Aspects of Adiabaticity in an Atomic BEC

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The successful generation of trapped atomic Bose-Einstein condensates in laboratories[1] has opened a new domain of atomic and optical physics, and challenging phenomena continue to be reported regularly through international journals. Theoretical studies of the phenomena, however, are traditionally based on the mean-field theory due to Bogoliubov[2], which was later extended by Gross and Pitaevskii[3] into the celebrated GP equation, whose basic premise is that the system behaves as a blob of matter representable by a field. This picture has proven so remarkably successful that most observed phenomena are indeed largely accountable by this equation, and conversely no theory thus far has been developed tangentially to this view. Despite all the success, there is one aspect of the BEC to which no complete treatment, to my knowledge, has been given, that is the free expansion used by experimentalists to achieve the optically resolvable image size of a BEC. Our investigation begins with the mystery that at least the density profile of a BEC remains intact during its expansion, inclusive of even the lattice structure of quantized vortices[4]; it suggests that some invariants, possibly adiabatic, might exist. In part, such an expectation is supported by the hyperspherical works of reference [5] and of [6]. Departing from the field theoretic picture, or rather from the independent particle representation, suffers serious and technically insurmountable disadvantages owing to the sheer number of particles involved. Our discussions to follow will thus keep the field-theoretic language and treatment closely in mind, though the pursuit of invariants, if any, naturally favors the framework based on the invariants themselves. As a specific candidate, we take the hyperradius, namely the effective size of the system, which corresponds to the well-defined optical image size when the system is expanded. Close to the ground state, this parameter's comportment suggests its connection with the single particle excitations. Anyhow, well-known from classical mechanics, this variable emerges in the context of the virial theorem, and is closely related to the uncertainty principle.

There seem to exist two working definitions of the hyperradius, each pertaining to a specific physical limit. One is the standard definition as employed in references [5] and [6] suitable for representation of single-particle excitations, and the other the local mean-square radius of the GP ground state solution pertinent to the evolution of collective modes. Leaving the details to the meeting, I wish to summarize highlights of our preliminary investigation.

1) The hyperspherical single-channel zero-th order approximation accounts for the monopole, dipole, and quadrupole excitations once the trap's isotropy is broken even infinitesimally.

2) The coupled GP equation for single-particle excitations may be solved by the adiabatic diagonalization method. Thus introduced are the local creation and annihilation operators. The hyperspherical single-channel zero-th order approximation yields the single-particle wave functions of low-energy excitation whose rough features resemble those of the GP solutions.

3) Collective oscillations as well as free-expansion of the BEC gas proceed largely adiabatically with respect to the local-mean square radius of the GP ground state solution so that the local particle density is a function of this single parameter. The wavepacket then retains its minimum uncertainty character. The retrieval of the phase should require additional, but not terribly strenuous, effort.

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