Ion dynamics and oscillation frequencies in a linear combined trap

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(Received 24 August 2000; accepted for publication 4 December 2000)

Ion traps have been pivotal in opening new frontiers for the precision spectroscopy of stable ions. We report on the demonstration of an additional ion trap: the linear combined trap. This device is particularly well suited for trapping ions with unstable nuclei, due to its large range of stability parameters that facilitates external injection online to an accelerator. The motion of an ion in such a linear combined trap is investigated theoretically and experimentally. In the trap ions oscillate harmonically in the radial direction and move nearly harmonically between fixed boundaries along the longitudinal axis. The presence of a homogeneous magnetic field and the applied dc and rf electric fields, lead to a set of coupled Mathieu equations. Their approximate solutions exhibit motional frequencies, which are combinations of shifted macromotion frequencies and the cyclotron frequency. The dependence of these motional frequencies on the applied trapping fields was studied in detail. For the measurements we used small clouds of laser cooled \(^9\)Be\(^+\) ions and crystallized \(^9\)Be\(^+\) ions. The observed oscillation frequencies are compared to both the results of zeroth order analytical solutions and to numerical simulations in which the frequency spectrum was obtained from the ion orbits by using the fast fourier transform formalism. The various motional resonances were experimentally recorded by applying a weak dipole excitation field to one of the trap electrodes, and by simultaneously observing at resonance the changes in the fluorescence intensity. Depending on the detuning of the cooling laser the ions gain energy during the motional excitation process in such a way that they are shifted further in or out of optical resonance by the Doppler effect. This leads to either positive or negative ion motion signals at the various ion oscillation frequencies. © 2001 American Institute of Physics. [DOI: 10.1063/1.1345514]

I. INTRODUCTION

In a combined trap we can store charged particles by superposing time-independent dc and oscillatory quadrupole potentials onto a homogeneous magnetic field. Such a combined trap can be considered to be a rf trap \(^1\)–\(^4\) and a Penning trap \(^5\)–\(^7\) in a single device. The symmetrical combined trap has been known for some time. \(^8\)–\(^13\) In it the ions move in a three-dimensional potential well. However, following the recent developments of linear traps \(^14,15\) we have proposed \(^16\) and built a linear combined trap. In this article measurements of the characteristics of the ion oscillation frequencies performed for this device are presented. In addition we derive an approximate theory that analytically describes the ion dynamics. The linear geometry provides a long potential valley, which is harmonic along the \(x, y\) directions, and near harmonically closed on both ends at suitably chosen distances along the \(z\) axis. In this way the linear trap is the ideal device for laser cooling many stored ions simultaneously, so that at low densities they arrange themselves in single file along the axis of the trap at the bottom of the potential valley. \(^17,18\) The combined trap has all the features of the rf and Penning traps, and, in addition, the possibility to simultaneously store particles with a large mass difference. When reducing either the magnetic field or the rf and dc electric fields the combined trap can be transformed into a pure rf trap or a Penning trap-like device.

Our main application of the linear combined trap is to make a precise measurement of the nuclear magnetic moments of the various isotopes of Be\(^+\) ions by laser-rf triple resonance spectroscopy. \(^19,20\) These measurements will be described in a forthcoming article. For high accuracy a strong magnetic field is required. \(^21,22\) For such measurements the combined trap is superior to the Penning trap due to the

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better signal to noise ratio as a consequence of the possible strong concentration of the cooled ions along the trap axis. Also, external injection is favored for the combined trap since it has a larger stability region than other storage devices. The combined trap also has ideal geometrical matching and good coupling for an upstream ion beam guide by which radioactive ions can be transported to the trap from an isotope separator. This is particularly useful when radioactive ions produced online at a nuclear accelerator need to be studied. In this context, the nuclear moments, as well as their distributions in a nucleus, can be determined.

This article is divided into seven parts. After the introduction we find in Sec. II an approximate analytical solution for the equations of motion of ions stored in a linear combined trap using a set of coupled Mathieu equations. Next, in the experimental section we described the apparatus and the measurement procedure using cooled clouds of ions and crystallized ions. In Sec. IV the observation of the various ion oscillation frequencies and their dependence on the operational parameters are given. Section V presents the numerical simulation of the ion dynamics. Here the fast Fourier transform (FFT) formalism yields the ion oscillation frequencies from the calculated ion orbits. In Sec. VI we compare the results of the measurements with theory and the simulations, and discuss the occurrence of spurious motional resonances. Finally, the different modes of the trap operation are related to each other in Sec. VII.

II. THEORY

A. Ion motion

Figure 1 displays the geometrical arrangement and segmentation of a linear combined trap into three sections, which carry the same rf voltage, but which can be individually biased to restrict the motion of the ions along the trap axis in the center section. The basic equation of motion of a charged particle of mass \( m \) and charge \( e \) in the trap is

\[
m\ddot{r} = -e\nabla \Phi + e\dot{r} \times B.
\]

The ions move under the influence of a magnetic field \( B \) and an electric field \(-\nabla \Phi\). The electric field is produced by applying the voltage \( \Phi_0(t) = \Phi_{dc} - V_{ac} \cos \Omega t \) between the adjacent electrodes of four cylindrical electrodes and the bias voltage \( V_z \). In the region near the midpoint of the trap axis, the potential \( \Phi \) can be approximated

\[
\Phi(x,y,z,t) = \Phi_0(t) \frac{x^2 - y^2}{2r_0^2} + \frac{\alpha V_z}{z_0} \left( z^2 - \frac{x^2 + y^2}{2} \right).
\]

Here \( 2r_0 \) is the separation of opposite electrodes, \( z_0 \) is half the length of the central region, and \( \alpha \) is a geometric factor of the trap. The magnetic field \( B \) is homogeneous and oriented along the \( z \) axis according to \( B = (0,0,B_0) \). In this way the Lorentz force provides additional radial confinement.

The equation of motion of an ion is

\[
\ddot{x} = -\frac{e}{mr_0^2} (V_{dc} - V_{ac} \cos \Omega t)x + \frac{eAV_z}{mz_0^2} x + \frac{eB_0}{m} y,
\]

\[
\ddot{y} = \frac{e}{mr_0^2} (V_{dc} - V_{ac} \cos \Omega t)y + \frac{eAV_z}{mz_0^2} y - \frac{eB_0}{m} x,
\]

\[
\ddot{z} = -2 \frac{eAV_z}{mz_0^2} z.
\]

The \( z \) equation describes the simple harmonic motion along the longitudinal \( z \) axis. In the transverse \( x \) and \( y \) directions the ions are harmonically bound. The motional equations can be rewritten in the form of coupled Mathieu differential equations

\[
\frac{d^2x}{d\tau^2} + \left( a - \frac{b}{2} - 2q \cos 2\tau \right)x - 2g \frac{dy}{d\tau} = 0,
\]

\[
\frac{d^2y}{d\tau^2} + \left( a + \frac{b}{2} - 2q \cos 2\tau \right)y + 2g \frac{dx}{d\tau} = 0,
\]

\[
\frac{d^2z}{d\tau^2} + bz = 0,
\]

with the dimensionless parameters

\[
\tau = \frac{1}{2} \Omega t, \quad a = \frac{4eV_{dc}}{mz_0^2 r_0^2}, \quad q = \frac{2eV_{ac}}{mz_0^2 r_0^2},
\]

\[
b = \frac{8eAV_z}{mz_0^2}, \quad \omega_e = \frac{eB_0}{m}, \quad \omega_c = \frac{\omega_e}{\Omega}.
\]

The equation along the \( z \) axis is independent from the transverse motion and can be solved separately

\[
z = Z_1 \exp(i\sqrt{b}\tau) + Z_2 \exp(-i\sqrt{b}\tau)
\]

\[
= Z_1 \exp(i\omega_c t) + Z_2 \exp(-i\omega_c t),
\]

where the \( z \) motion frequency is given by

\[
\omega_c = \frac{1}{2} \sqrt{b} = \frac{1}{2} \sqrt{\frac{2eAV_z}{mz_0^2}}.
\]
In the special case that \( a = g = b = 0 \) and \( q \neq 0 \), the equations along the transverse directions reduce to the uncoupled Mathieu equations; the solution of which is the well-known secular oscillation of the ion motion:

\[
x(\tau) = \sum_{n=-\infty}^{\infty} x_n(\tau) = A_0 \sum_n C_{2n} \exp[i(\beta_0 + 2n)\tau] + A'_0 \sum_n C_{2n} \exp[-i(\beta_0 + 2n)\tau],
\]

\[
y(\tau) = \sum_{n=-\infty}^{\infty} y_n(\tau) = B_0 \sum_n C_{2n} \exp[i(\beta_0 + 2n)\tau] + B'_0 \sum_n C_{2n} \exp[-i(\beta_0 + 2n)\tau],
\]

with \( \beta_0 = \sqrt{q^2/2} \). For the zeroth order approximation of Eq. (4), if \( a = g = b = 0 \) we replace the secular motion by harmonic dynamics in the pseudopotential according to

\[
\frac{d^2x}{d\tau^2} = -(2q \cos 2\tau) x = -\beta_0^2 x,
\]

\[
\frac{d^2y}{d\tau^2} = -(2q \cos 2\tau) y = -\beta_0^2 y,
\]

yielding when \( \beta_0 \) is inserted into Eq. (4)

\[
\frac{d^2x}{d\tau^2} + \left( \beta_0^2 + a - b/2 \right) x - 2g \frac{dy}{d\tau} = 0,
\]

\[
\frac{d^2y}{d\tau^2} + \left( \beta_0^2 - a - b/2 \right) y + 2g \frac{dx}{d\tau} = 0.
\]

Assuming that the solutions have the form

\[
x = A \exp(i\beta \tau) \quad \text{and} \quad y = B \exp(i\beta \tau).
\]

and substituting above, we obtain

\[
\begin{bmatrix}
-\beta^2 + \beta_0^2 + a - b/2 & -i2g\beta \\
+i2g\beta & -\beta^2 - \beta_0^2 - a - b/2
\end{bmatrix}
\begin{bmatrix}
A \\
B
\end{bmatrix} = 0.
\]

The characteristic equation of this matrix can be solved and is given by

\[
\beta^2 = \beta_0^2 - \frac{b}{2} + 2g^2 \pm \sqrt{4g^2 \left( \beta_0^2 - \frac{b}{2} + g^2 \right) + a^2}
\]

\[
\approx \frac{q^2}{2} - \frac{b}{2} + 2g^2 \pm \sqrt{2g^2(q^2 - b/2 + g^2) + a^2}.
\]

Substitution of Eq. (13) into Eq. (11) yields the \( x \) and \( y \) coordinates of an ion orbit and two motional frequencies. We label these frequencies \( \omega_h \) (high) and \( \omega_l \) (low). It follows

\[
\omega_{h,l} = \frac{\beta \Omega}{2}
\]

\[
= \left[ \omega_z^2 - \frac{\omega_c^2}{2} \pm \sqrt{\omega_z^2 - \frac{\omega_c^2}{2} + \omega_s^2} \right]^{1/2} + \omega_s^2/4
\]

with

\[
\omega_0 = \frac{\beta \Omega}{2} \approx \sqrt{\frac{q^2}{2}}, \quad \omega_s = \frac{\sqrt{|a|}}{2}.
\]
the ion fluorescence in the measurement section by a suitably applied bias voltage $V_{z2}$. The trap is filled with $^{9}\text{Be}^{+}$ ions (natural isotopic abundance 100%) by evaporating and ionizing Be atoms with a pulsed YAG laser with an energy of 15 mJ/pulse and a pulse duration of 6 ns from a beryllium disk located next to the central loading section. For dipole excitation of the motional frequencies, a small excitation rf voltage, $V_{s}\cos 2\pi f_{s}t$, was applied to one of the ion trap electrodes where $V_{s}$ was about 0.01 V.

The maximum field of the superconducting magnet was 1 T. The magnetic field was calibrated with a small nuclear magnetic resonance probe. The magnetic field homogeneity over the region of the trapped ions of about 1 mm$^3$ was $|\Delta B/B| = 5 \times 10^{-7}$. Since the bore in the magnet was at room temperature, it enabled us to insert and retract the glass tube containing the ion trap at all times.

A beam of single frequency tunable laser radiation of typically 100 $\mu$W power was focused to a waist of about 100 $\mu$m at the location of the ions. For our experiments the strong resonance transition from the $^{2}\!S_{1/2}$ ground state to the $^{2}\!P_{3/2}$ excited state with radiation at $\lambda = 313$ nm was employed. A partial energy level diagram of $^{9}\text{Be}^{+}$ is depicted in Fig. 4. In this diagram the hyperfine structure splitting (HFS) is neglected. For cooling, the laser frequency was slightly red detuned and either $\sigma^{-}$ or $\sigma^{+}$ radiation was used. A small angle of about 4° between the laser beam and the trap axis was chosen to efficiently remove energy from all degrees of freedom of the ion cloud and to produce few ion crystals. The sensitivity was such that the motional resonances of single ions could be observed. The ion fluorescence was imaged by a combination of achromatic lenses through a side hole in the superconducting magnet onto the cathode of a position resolved single photon counting camera (Hamamatsu type PIAS-TI).

B. Measurement procedure

All measurements of the ion motion spectrum were carried out on laser cooled ions, although some spectra are obtained on ion clouds, others on ion crystals. The motional state of the ions was dependent on the laser detuning $\delta$, the power of the cooling laser, and the number of ions in the trap. Figure 5 displays a sequence of fluorescence spectra as a function of laser detuning $\delta$. For each scan, the number of stored ions is different and decreases from trace (a) to (c). For a small enough ion cloud (typically less than 100 ions) the spectrum exhibits the characteristic phase transition from a cloud state to a crystalline state. The wide structures on the left (a), (b), and (c) represent cloud spectra, while the high peak on the right of (c) indicates the spectrum of an ion
crystal. The minimum between the two kinds of spectra corresponds to the phase transition, where the disordered cloud state changes instantaneously to the ordered crystalline state. For the example shown, the peak intensity of the fluorescence in the crystal state is about ten times the peak intensity in the cloud state.

Next the ion motion frequencies were excited by sweeping an additional rf driving field through resonance. The driving voltage was applied to one central trap electrode. For these measurements, the dipole mode of excitation was used. Figure 6(a) shows the location of the various ion motion resonances and their dependence on the magnetic field. Depending on the value of the magnetic field, one major resonance namely \( f_{h} \) shifts to higher oscillation frequencies, whereas the other one, \( f_{l} \), shifts to lower ones \( (f_{h,l} = \omega_{h,l}/2\pi) \). Depending on the detuning \( \delta \) and temperature of the ions, the observed changes in fluorescence intensity are either negative or positive. In a typical experiment, after loading the trap with a large number of ions the laser is initially detuned by about 700 MHz to the red side of the resonance. In this case, a cloud spectrum is observed. When the ions are resonantly heated by driving their macromotion, the cloud both expands and becomes detuned by the increasing Doppler width. This causes the fluorescence to decrease and a motional resonance is observed as a dip. It should be noted that, in general, the ions are not lost in this process since the fluorescence returns to the same level as it had before the motional resonance was excited. Outside a motional resonance the damping of the ion motion by the red detuned laser forces the cloud back to the original cooled state. While taking the various sequences, care was taken to ensure that the magnetic field of the superconducting magnet had stabilized before the next measurement was carried out. All spectra were observed during one measurement cycle of about 10 h. During this time, a few ion loadings were carried out. Only the number of stored ions changes decreasing from several hundred in the top traces to just a few ions in the bottom traces. To take the measurements on just a few ions (last few traces), the laser detuning was reduced to be only 200 MHz towards the red side. At this value an ion crystal just beyond the phase transition point having only weak fluorescence was produced. In this situation, resonant heating at a macromotion frequency transforms the ion crystal to a cloud state which has a higher fluorescence intensity and a fluorescence peak indicates the motional resonance.

IV. EXPERIMENTAL RESULTS

The measured magnetic field dependence of \( f_{h} \) and \( f_{l} \) is plotted in the Fig. 6(b). The data was compiled from measurements such as shown in Fig. 6(a). At times some additional weak resonance signals appear in the spectra. Although they display a similar field dependence, they are at different frequencies than the \(^{9}\text{Be}^{+}\) resonance. (We will discuss the origin of these spurious resonances in Sec. VI.C.)

The applied trap operation parameters were \( \Omega/2\pi = 7.30 \text{ MHz} \), \( V_{d}^{\text{pp}} = 100 \text{ V} \), \( V_{dc}^{\text{pp}} = 0 \). For the present unbalanced mode of applying the driving voltages, we had to use voltages with respect to the trap center, and also effective voltages \( V_{ac} \) and \( V_{dc} \). The effective voltages are the values seen by the stored ions and are slightly different from the applied voltages. The distinction is necessary for \( V_{dc} \) due to the use of circular and nonhyperbolic shaped trap electrodes as required by theory. The effective \( V_{dc} \) is the sum of applied \( V_{d}^{\text{pp}} \) and a small correction. This correction is the contact
potential contribution, which arises when a minute beryllium film deposits on the trap electrodes during the laser ablation while loading the trap with ions. The extra loading section minimized this effect, but could not completely eliminate it. The calibration factors for the effective voltages were obtained by measuring the radial ion oscillation frequencies when the trap was operated at a negligibly weak magnetic field of $B = 4 \times 10^{-3}$ T.

The calibration frequencies $f_{h,l}^{0}$ are the zero magnetic field intercepts displayed in Fig. 6(b) and were measured as a function of $V_{ac}$ and $V_{dc}$ as shown in Figs. 7 and 8, respectively. The q and a factors are obtained from the dependence of the average frequency $f_{av}^{q} = (f_{h} + f_{l})/2 = \sqrt{2} q \Omega/8 \pi$ and the difference frequency $\Delta f^{q} = f_{h}^{0} - f_{l}^{0} = a \Omega^{2}/16 \pi^{2} f_{av}^{q}$. In this approximation, the variation of the closing voltage $V_{z}$ over the trapped ion cloud can be neglected due to the smallness of the cloud. When inserted into Eq. (5) the effective voltages $V_{ac}$ and $V_{dc}$ follow. This procedure yields $V_{ac} = 0.95 V_{app}$ for our trap.

Figure 8 gives the relation $V_{dc} = V_{app} - 0.1$ V. The -0.1 V contribution is mainly due to the contact potential. Since the contact potential changes slowly over time periods of days, this value is not the same in all measurements. The small nonzero intercepts in Figs. 7 and 8 are due to the presence of a weak magnetic field and in Fig. 8 correspond with 8.3 kHz to the cyclotron frequency of $^{9}$Be$^{+}$ at $4 \times 10^{-3}$ T.

V. SIMULATION OF THE ION MOTION DYNAMICS

The equation of the motion for a single $^{9}$Be$^{+}$ in a combined trap was also solved numerically. For the general case in which the magnetic field is inclined, $B = B_{0}(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, and inserting this into Eq. (3), the equations of motion become

$$\dot{x} = -\frac{e}{m r_{0}^{2}} (V_{dc} - V_{ac} \cos \Omega t) x + \frac{e a V_{z}}{m z_{0}^{2}} x$$
$$+ \frac{e B_{0}}{m} (\dot{y} \cos \theta - \dot{z} \sin \theta \sin \phi),$$
$$\dot{y} = + \frac{e}{m r_{0}^{2}} (V_{dc} - V_{ac} \cos \Omega t) y + \frac{e a V_{z}}{m z_{0}^{2}} y$$
$$+ \frac{e B_{0}}{m} (\dot{z} \sin \theta \cos \phi - \dot{x} \cos \theta),$$
$$\dot{z} = - \frac{2 e a V_{z}}{m z_{0}^{2}} + \frac{e B_{0}}{m} (\dot{x} \sin \phi - \dot{y} \cos \phi) \sin \theta.$$
where we have set $\omega_{l,h} = 2\pi f_{l,h}$. FFT spectra of the dominating zeroth order for various cases are depicted in Fig. 10. When the magnetic field is absent and $V_{dc} = 0$ the two ion oscillation frequencies coincide (a). With increasing magnetic field the two motional frequencies split and are shifted toward higher and lower frequencies respectively (b). When $V_{dc} = 0.75 V$, $f_h$ and $f_l$ are already split in the absence of a magnetic field (c). The splitting increases with $V_{dc}$ and increases further as a function of increasing magnetic field.

VI. DISCUSSION OF THE RESULTS

Two theoretical approaches to calculate the ion oscillation frequencies have been described, and experimentally these frequencies have also been observed. In this section we compare the experimental results with the two theoretical calculations.

A. Comparison of experiment with the numerical simulation and the analytical solution

The magnetic field dependence of the radial macromotion frequencies $f_h$ and $f_l$ obtained by the various methods are shown in Fig. 11(a). The solid lines were calculated using the analytical solution of Eq. (14), the symbols give the results of the experiment and the numerical simulations. It can be seen that the results of the three methods give somewhat different values. The differences $\delta f$ of the macromotion frequencies for the experimental and analytical values on one side with the simulation results on the other side are compiled for $f_h$ in Fig. 11(b) and for $f_l$ in Fig. 11(c) as a function of the magnetic field. The agreement is best between experiment and simulation.

Figure 12 indicates the contribution due to a nonzero bias voltage to the motional frequencies by plotting the frequency difference for $V_{dc}=0.75 V$ and $V_{dc}=0$ versus magnetic field. It can be seen that for weak magnetic field ($B<0.2 T$), $f_h$ and $f_l$ have a strong dependence on $V_{dc}$, and that in the strong field region ($B>0.5 T$) $f_h$ approaches the zero dc field value but $f_l$ retains a small finite influence. This behavior is expected, since for $f_h$ the magnetic field effect adds, but for $f_l$ it subtracts from the motional frequencies without magnetic field.

In order to investigate where the approximate analytical solution deviates most from the more accurate simulation results we have plotted, in Fig. 13, the ion motion frequency difference $\delta f$ for a fixed magnetic field as a function of the stability parameter $q$. $\delta f_h$ is particularly large for large amplitudes $V_{dc}$ of the driving field at frequency $\Omega$, or more generally large values of the stability parameter $q$. This indicates the well known fact that the zeroth order solution results of the Mathieu equation are not sufficient for large values of $V_{dc}$ or $q$ and in this case the solutions of all order have to be employed simultaneously to get the correct result. For the case of large $q$ the macromotion frequency $f_h$ is no longer small compared with the driving frequency $\Omega$ and the cyclotron frequency $f_c$, and therefore the condition for the dominance of the zeroth order solution is no longer fulfilled. For the same reason $\delta f$ is large for small values of $\Omega$.

B. Motion and oscillation frequencies in the $z$-direction

In the experiment a dc bias voltage $V_z$ is applied to the outer trap segments to confine the ions along the $z$ direction.
them in the experiment. The results of the numerical simulations using Eq.
we have numerically calculated the effects of these condi-
spect to the direction of the magnetic field. In the following
\begin{equation}
V
\end{equation}
operating parameters
\begin{equation}
\text{components of the corresponding two coupled frequencies}
\end{equation}
\begin{equation}
\text{field since they are strongly coupled. We labeled the}
\end{equation}
\begin{equation}
du
\end{equation}
In addition, in the present experimental arrangement, the trap
electrodes were inclined at an angle \( \theta \) of about 4° with re-
spect to the direction of the magnetic field. In the following
we have numerically calculated the effects of these condi-
tions using Eq. (20). At the same time we have also observed
them in the experiment. The results of the numerical simu-
lation are shown in Fig. 14 for the angles \( \theta = 4° \) and the trap
operating parameters
\begin{equation}
V_{dc}=100 \text{ V}, \ V_{ac}=75 \text{ mV}, \ \Omega/2\pi =6.63 \text{ MHz}, \text{ and } B=0.4066 \text{ T.}
\end{equation}
The effect of the dc bias voltage \( V_{z} \) was included by setting
\begin{equation}
f_{1}(V_{z})=95 \text{ kHz. The two}
\end{equation}
frequencies
\begin{equation}
f_{1} \text{ and } f_{2} \text{ have similar values in the strong mag-
netic field since they are strongly coupled. We labeled the}
\end{equation}
components of the corresponding two coupled frequencies
\begin{equation}
f_{11} \text{ and } f_{12}. They only appear when the magnetic field is}
inclined. There is also a dependence of the two coupled fre-
frequencies on \( V_{ac}. Figure 15(a) contains the experimental val-
ues of \( f_{11} \) and \( f_{12} \) measured for different values of \( V_{ac}. The
simulation results are also indicated for the inclination angle
\begin{equation}
\theta = 4° \text{ of the magnetic field and agree well with the experi-
mental results.}
\end{equation}

C. Spurious resonances

The identification of the observed resonances is made by
their frequency values and their dependence on the operating
parameters. In this context we have intermittently observed
two series of weaker resonances at unexpected frequencies of
about \( \omega_{1}/3 \) and \( \omega_{1}/3 \) [labeled \( f_{h} ' \) and \( f_{l} ' \) in Fig. 6(b)]. These
resonances can be explained by the presence of impurity
ions, which have a mass of about three times the beryllium
mass and therefore are probably caused by Al or Be\textsuperscript{3+} clus-
ters. It should be noted that also \( f_{1} ' \) coincides with \( f_{1} \) of
\begin{equation}
^{9}\text{Be}. The spurious resonances make themselves noticeable in
the Be\textsuperscript{+} fluorescence through sympathetic heating\textsuperscript{28} of
Be\textsuperscript{+} ions by motionally excited impurity ions. In this process
kinetic energy is transferred to the Be\textsuperscript{+} ions from the reso-
nantly excited impurity ions through Coulomb collisions.
The sympathetic heating effect is largest when only a few
single ions and a few impurity ions in the crystalline state are
concentrated in close proximity near the ion trap axis. This is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig11.png}
\caption{(a) Field dependence of the ion motion frequencies observed by experiment and calculated in the numerical simulation and the analytical solution. (b) and (c) Comparison of the analytical results with the experimental and numerical FFT results. The deviation in ion motion frequencies \( \delta f_{\circ} \) vs magnetic field is shown. (From the results in the present experiment we observed \( f_{l} ' =95 \text{ kHz} \) and \( f_{h} ' =320 \text{ kHz} \) at \( B_{0}=0.004 \text{ T} \), \( \Omega/2\pi =7.30 \text{ MHz}, \ V_{dc}=100 \text{ V}, \ V_{ac}=0, \text{ and } V_{m}=5 \text{ V}. It can be estimated that } \ V_{ac} =95 \text{ V}, \ V_{dc}=4.53 \text{ V}, \text{ and } f_{2}(V_{z})=105 \text{ kHz, therefore these values are used in the simulation and the analytical solution. In the simulation the inclination angles } \theta = 4° \text{ and } \phi = 45° \text{ were taken into consideration.})}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig13.png}
\caption{Comparison of the numerical FFT results with the analytical results. The difference in calculated ion motion frequencies versus the stability parameter \( q = q(V_{ac}, \Omega). (V_{ac} \text{ dependence: } V_{dc}=0, \ \Omega/2\pi =8.0 \text{ MHz}, \ B_{0} =1 \text{ T, } \theta = \phi = 0. \text{ } \Omega \text{ dependence: } V_{ac}=100 \text{ V}, \ V_{dc}=0, \ B_{0}=1 \text{ T, } \theta = \phi = 0.)}
\end{figure}
evident from Fig. 6 where these resonances appear only in the lower traces when only a few ions are stored.

VII. MODES OF OPERATION

It is possible to operate the combined trap in two extreme regions, and instructive to consider these two cases in detail. Near zero magnetic field, the operation is close to that of a rf trap whereas near zero ac voltage the operation is close to that of a Penning trap. Earlier, for the case of the symmetrical trap a particular example in which the magnetron motion vanishes has been discussed. We have also studied these extreme regions of the linear combined trap. The measurements and results of the calculations are compiled in Fig. 16. The observations at near zero magnetic field demonstrate that in this limit the ion oscillation frequencies are made up of two harmonic oscillations about the trap center with a slow drift motion around the field lines superimposed on them, yielding and . In addition motion. In this case the radial confinement is dominated by the rf fields. In the other extreme case of the near Penning trap mode, the low amplitudes of the driving rf field contribute very little to the radial trapping. We observed neither fluorescence nor motional resonances in this region. This fact can be explained by considering Fig. 15, which illustrated the results of a calculation. Similar to the case of the circular combined trap, there is an unstable region around the point where the direction of the magnetron rotation changes. In our case, of the linear combined trap, the additional static quadrupole field in the radial direction due to the presence of a residual causes an even broader unstable region. In the present trap, typical values of the residual are about 100 mV. This fact makes even trap operation in the pure Penning trap mode difficult.

FIG. 14. (a) FFT spectra without inclination, (b) FFT spectra with the inclination angle . The motions of the , , and directions are coupled. (V = 100 V, V = 0, / = 7.04 MHz, B = 0.4066 T, = 45°, and (V,) = 95 kHz.)

FIG. 15. (a) Measured and calculated inclination effect of the magnetic field on the and versus V. Different ion trajectories (b) V = 5 V and (c) V = 30 V. Note that the rotational direction of changes. (Experiment: V = 0.95 V, V = 0, / = 6.63 MHz, B = 0.4066 T, V = 5 V. Simulation: / = 6.63 MHz, V = 75 mV, B = 0.4066 T, = 45°, (V,) = 95 kHz.)

FIG. 16. Evolution of the ion motion frequencies with the magnetic field (left) and with the ac driving voltage (right). For zero magnetic field the Paul trap mode occurs. For zero ac voltage the near Penning trap results [Note the different maximum values of the magnetic field, which is B_max = 0.6 T (left) and B = 0.4066 T (right)].
VIII. SUMMARY

We have demonstrated a trapping technique that is particularly well suited for catching externally generated ions. The analytical and numerical models explain the observed oscillation frequencies. The importance of the coupling of the various degrees of freedom of the ion motion is reflected by the shifts of the oscillation frequencies in the different magnetic field regions, in the various modes of operation, and the possible orientations of the magnetic field.

ACKNOWLEDGMENTS

The authors acknowledge valuable discussions with X. Zhao and G. Agnolet.

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