生命情報学特別講義

－デジタル画像と定量化－
その4：ノイズ除去・画像復元・フィルタ

第４回講義
2011年8月3日～4日
伊都新キャンパス

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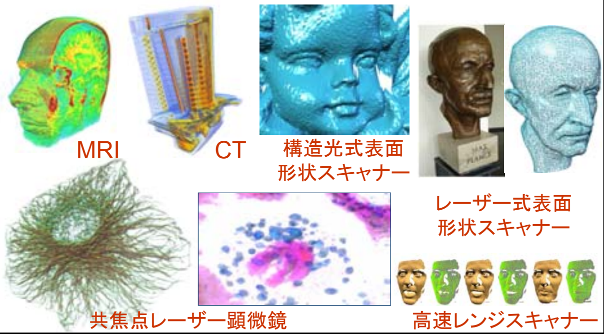
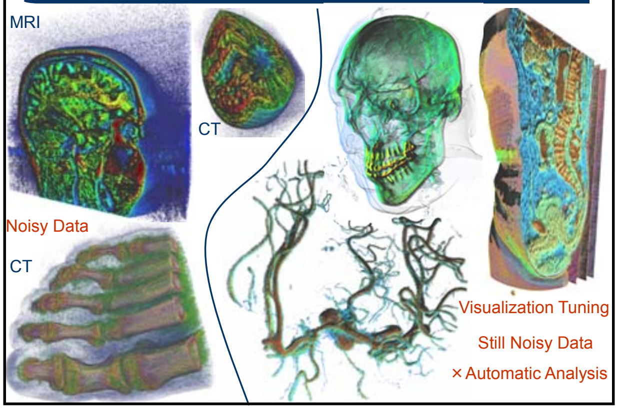
ノイズ除去・画像復元・フィルタリングの背景

- 自然科学では観察・観測による画像解析が重要である.
- 測定・計測に基づくCAD/CAM/CAEも注目されている.

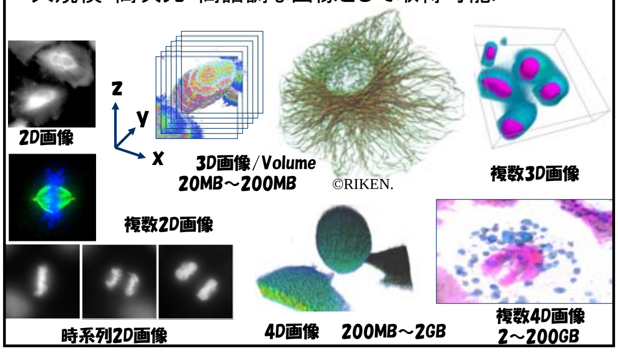


生物・医用画像



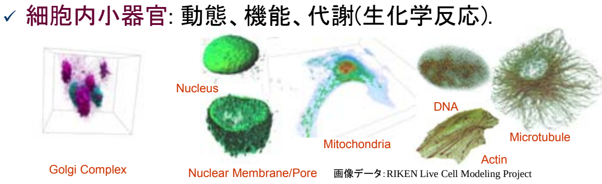
細胞内観察画像

- 共焦点レーザー顕微鏡の発達により, 細胞内部の構造を大規模・高次元・高諧調な画像として取得可能.

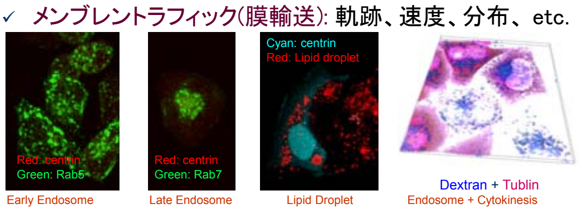


細胞内観察画像

- 細胞内小器官: 動態、機能、代謝(生化学反応).



- メンブレントラフィック(膜輸送): 軌跡、速度、分布、etc.



細胞内観察画像の特徴

- 共焦点レーザー顕微鏡＆蛍光観察自体が新しい.
- 莫大なデータサイズ＆時系列(高次元データ).
- 多チャンネルデータ(多色蛍光観察等による複数画像).
- Limited number of simultaneous observation.
- High sensitivity imaging generates noisy images !
- Dynamic & complex topology & geometry.

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フィルタの基礎

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線形フィルタ

✓ 線形フィルタ（畳みこみ和）

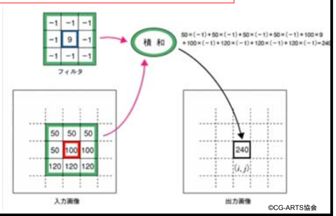
$$g(i,j)=\sum_{n=-w}^{w}\sum_{m=-w}^{w} f(i+m,j+n)h(m,n)$$

但し、 *h(m,n)*　　フィルタの係数

(2w+1)×(2w+1)　　フィルタの大きさ

✓ $f(i,j)$ は入力画像
✓ $g(i,j)$は出力画像
✓ $h(i,j)$はカーネル画像

この式に当てはまらないフィルタは、非線形フィルタ



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モザイクフィルタ

ある範囲を一色へ置き換える.

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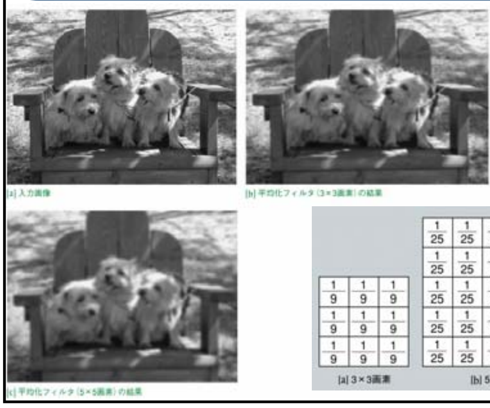
平均化フィルタ



ある範囲の平均.

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

[a] 3×3画素

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

[b] 5×5画素

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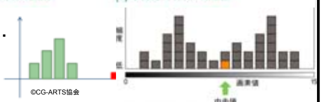
メディアン(中央値)フィルタ

✓ ある範囲の中央値に置換.
✓ 異常値を検出.
✓ エッジを(ある程度)保存.



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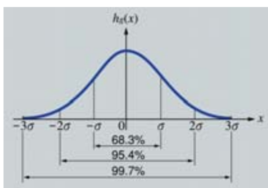
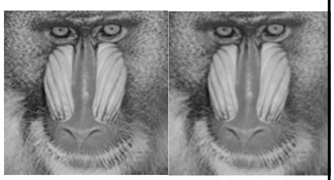
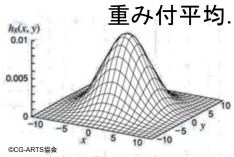
Gaussianフィルタ





Gauss関数の重み付平均.

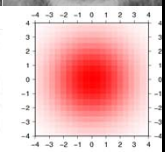


1/16	2/16	1/16
2/16	4/16	2/16
1/16	2/16	1/16

[a] 3×3画素

1/256	4/256	6/256	4/256	1/256
4/256	16/256	24/256	16/256	4/256
6/256	24/256	36/256	24/256	6/256
4/256	16/256	24/256	16/256	4/256
1/256	4/256	6/256	4/256	1/256

[b] 5×5画素



$$h_g(x,y)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

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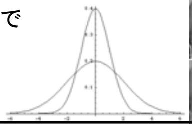
Gaussianスケールスペース

- ✓ 異なるσでスムージングすると、様々なレベルでの画像.
- ✓ σを"スケール"として、「異なるスケールの画像」.
- ✓ $t=s^2$として、次のようにガウスフィルタを一般化:

$$h_g(x,y;t)=\frac{1}{2\pi t}\exp\left(-\frac{x^2+y^2}{2t}\right)$$

- ✓ 世界は、スケールによって異なる構造をもつ、という考え方.
- ✓ σで偏微分する事でスケールの極値や変化率を図る.



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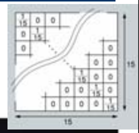
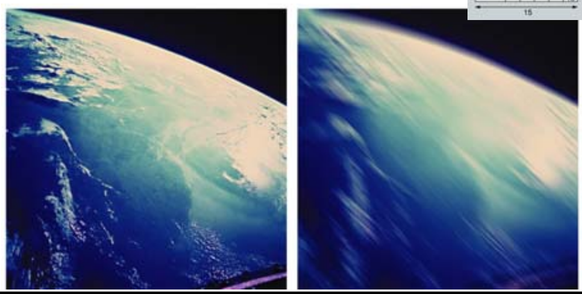
Anisotropic(異方性)フィルタ

- ✓ カーネル画像をデザインする事により特定方向の重み付平均化を行える.





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畳み込み:空間 VS 周波数

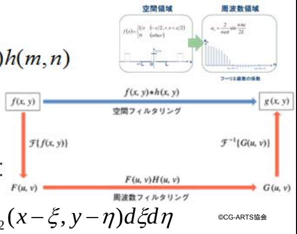
- ✓ 線形フィルタ(畳みこみ和):

$$g(i,j)=\sum_{n=-w}^{w}\sum_{m=-w}^{w}f(i+m,j+n)h(m,n)$$

畳み込み積分の離散版.

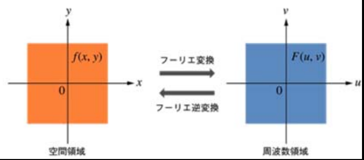
- ✓ 畳み込み積分(convolution):

$$f_1(x,y)*f_2(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f_1(\xi,\eta)f_2(x-\xi,y-\eta)d\xi d\eta$$



畳み込みは周波数領域では掛け算になる!

$$f*g=F^{-1}[F[f]F[g]]$$



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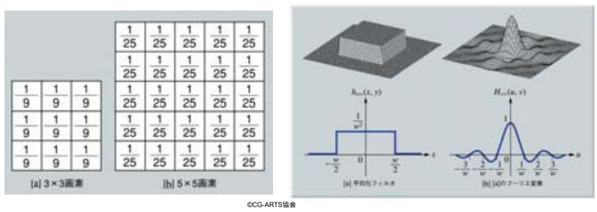
畳み込み:空間 VS 周波数2

$$h_{ave}(x,y)=\frac{1}{w^2}rect\left(\frac{x}{w},\frac{y}{w}\right)\qquad H_{ave}(u,v)=\frac{\sin\pi wu}{\pi wu}\frac{\sin\pi wv}{\pi wv}$$

$$rect(x,y)=\begin{cases}1 & |x|\le\frac{1}{2}かつ|y|\le\frac{1}{2}のとき\\0 & その他\end{cases}$$

平均化フィルタ

空間領域　　周波数領域



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ノイズ除去

一般的なノイズ
ー加算性白色ガウスノイズー

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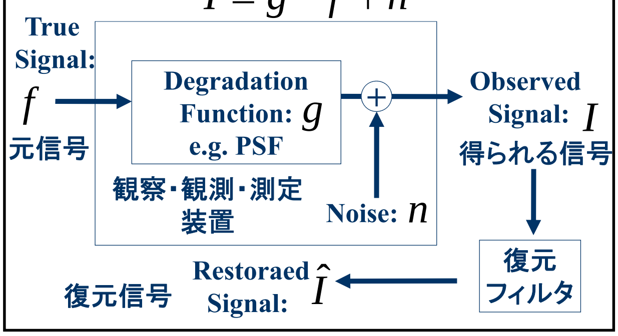
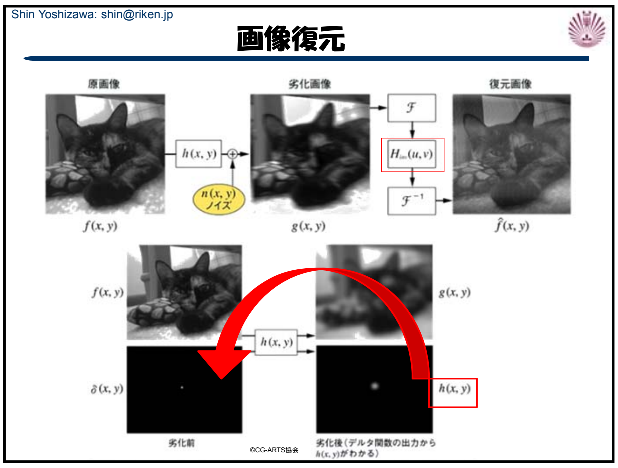
Noise ?

- 例えば加算ノイズモデル(Additive Noise Model)

$$I=g*f+n$$

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画像復元
原画像
劣化画像
復元画像
h(x, y)
n(x, y) ノイズ
f(x, y)
g(x, y)
f̂(x, y)
δ(x, y)
劣化前
劣化後（デルタ関数の出力から h(x, y)がわかる）
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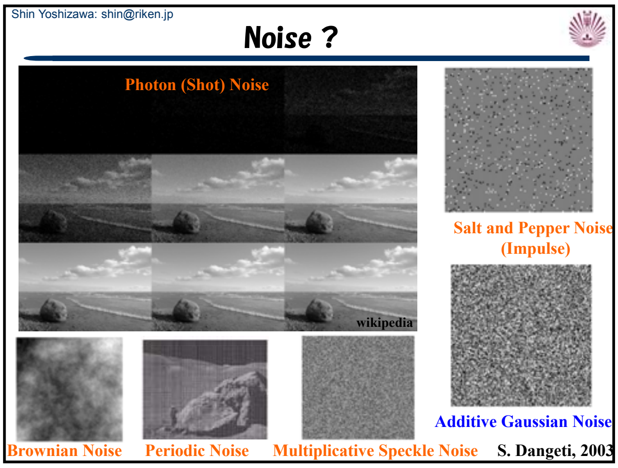
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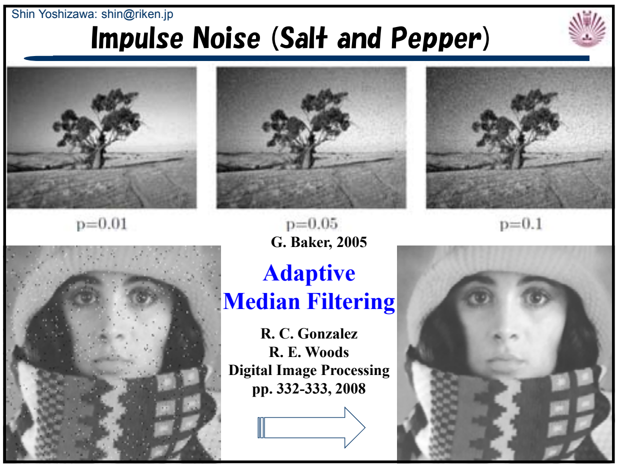
Noise ?
Photon (Shot) Noise
wikipedia
Salt and Pepper Noise (Impulse)
Additive Gaussian Noise
Brownian Noise
Periodic Noise
Multiplicative Speckle Noise
S. Dangeti, 2003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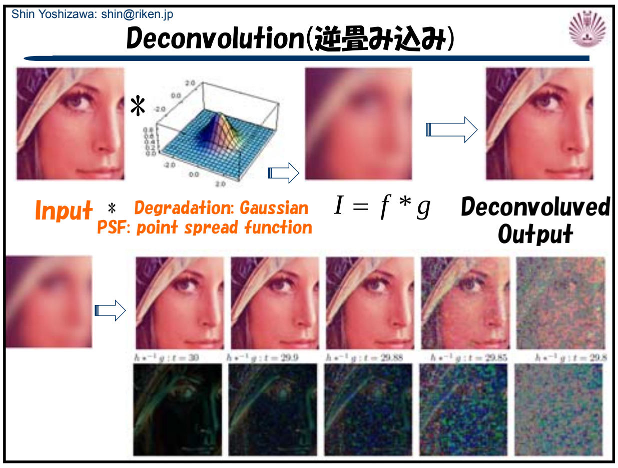
Impulse Noise (Salt and Pepper)
p=0.01
p=0.05
p=0.1
G. Baker, 2005
Adaptive Median Filtering
R. C. Gonzalez
R. E. Woods
Digital Image Processing
pp. 332-333, 2008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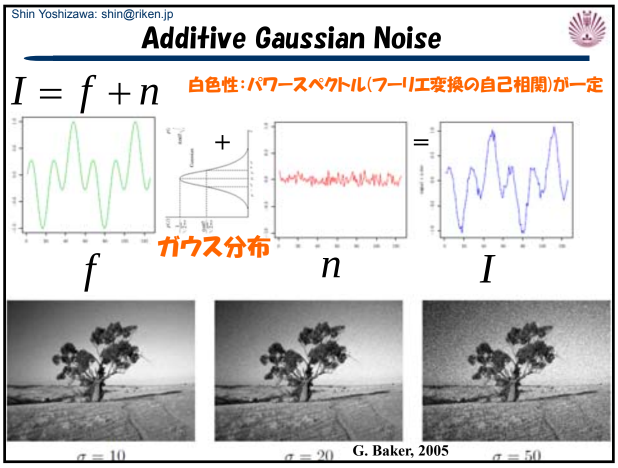
Deconvolution(逆畳み込み)
Input
Degradation: Gaussian
PSF: point spread function
I = f * g
Deconvoluved Output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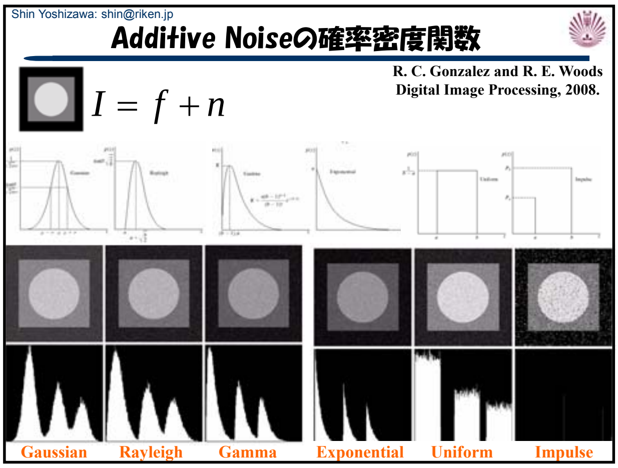
Additive Gaussian Noise
I = f + n
白色性：パワースペクトル(フーリエ変換の自己相関)が一定
ガウス分布
f
n
I
σ = 10
σ = 20
σ = 50
G. Baker, 2005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Additive Noiseの確率密度関数
I = f + n
R. C. Gonzalez and R. E. Woods
Digital Image Processing, 2008.
Gaussian
Rayleigh
Gamma
Exponential
Uniform
Impulse

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Wiener Filter

- ノイズ関数をNとすると、

$$G(u,v)=F(u,v)H(u,v)+N(u,v)$$

- 復元画像と原画像の誤差を最小にするような逆フィルター
  :Hが0に近くても発散しない.

$$H_w(u,v)=\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+|N(u,v)|^2/|F(u,v)|^2}$$

$$\boxed{H_w(u,v)=\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+\Gamma}}$$

通常は，NとFは不明なので，
適当な定数で近似.

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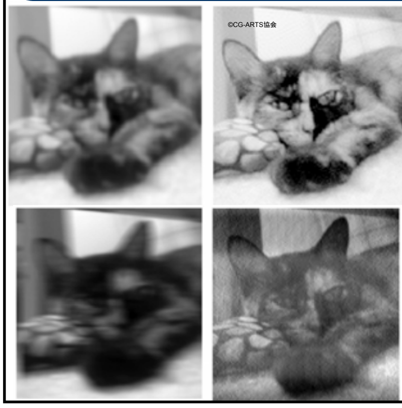
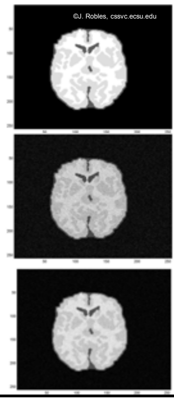
Wiener Filter2





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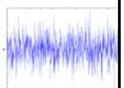
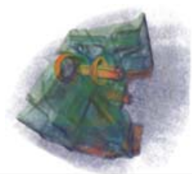
Additive White Gaussian Noise

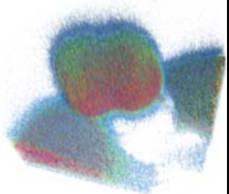
- 計測・観察によって得られるデータは通常ノイズを含む。
- 加算性白色ガウスノイズは自然界の雑音を良く近似する。
  - 最小二乗的に最適なノイズ除去法はWiener Filter.
  - 理想フィルタは原信号のパワースペクトルが必要…
  - 複雑な計測・観察データでは実用上「平滑化法」.



実測データにおける装置による雑音の例

平均曲率の曲面上表示





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特徴保存ノイズ除去フィルタ:平滑化 (Smoothing)

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Noise Reduction by Smoothing

- Smoothing as Convolution:　　Convolution Kernel: g

$$f*g=\int f(t)g(x-t)dt,\quad f,g\in C^\infty(\Re),\quad |x|\to\infty,\ f(x),g(x)\to 0.$$

Normalized Convolution:

$$\int f(t)g(x-t)dt/\int g(x-t)dt.$$

- Smoothing as Diffusion (Partial Differential Equations):

$$h(\mathbf{x},t)=\int f(\mathbf{y})g(\mathbf{x}-\mathbf{y},t)d\mathbf{y}=\frac{1}{(4\pi t)^{n/2}}\int f(\mathbf{y})G_{\sqrt{4t}}(|\mathbf{x}-\mathbf{y}|)d\mathbf{y},\quad G_a(b)=e^{-(b/a)^2}.$$

拡散方程式: $\frac{\partial h}{\partial t}=\Delta h=\mathrm{div}(\mathrm{grad}(h(\mathbf{x},t))),\ \mathbf{x}\in\Re, t>0, h(\mathbf{x},0)=f(\mathbf{x}).$

の基本解

$$\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\ldots+\frac{\partial^2}{\partial x_n^2},\quad \mathbf{x}=(x_1,x_2,\ldots,x_n)\in\Re^n.$$

$$\frac{1}{2}\int|\nabla h(\mathbf{x})|^2 d\mathbf{x}\to\min\Rightarrow\Delta h(\mathbf{x})=0$$

Gaussian Smoothing = Laplacian Smoothing =拡散・熱伝導方程式の解多様体(曲線、曲面、etc.)

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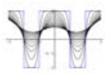
What is the Smoothing ?

- Smoothing as Variational Problems:

$$\frac{\partial h}{\partial t}=\Delta h,\quad t\to\infty\Rightarrow\Delta h=0.$$ with appropriate boundary condition.

Dirichletエネルギー最小化問題: $\int|\nabla h|^2\to\min.$



- Smoothing in Transformed Domain:

$$\frac{\partial u}{\partial t}=\Delta u,\quad u(x,0)=h(x),\quad 0\le x\le l,\quad u(0,t)=u(l,t)=0,\quad x,u(x,t)\in\Re.$$

Fourier解: $u(x,t)=\sum_{k=1}^{\infty}b_k\exp(-\frac{k^2\pi^2}{l^2}t)\sin\frac{k\pi}{l}x,\quad b_k=\frac{2}{l}\int_0^l h(y)\sin\frac{k\pi}{l}ydy.$

$t\to\infty$ 高周波ほど急激に減少。

Gaussian Smoothing = Laplacian Smoothing =拡散・熱伝導方程式の解多様体=Dirichletエネルギーの最小化=高モードFourier係数の0への置き換え=Low Pass Filter.

Convolution Kernel・関数変換の基底(wavelet等)に対応するPDE・変分問題。

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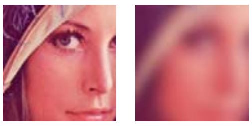
Smoothing in Image Processing

- Linear Diffusion, Gaussian Smoothing: Gabor 1960.

$$\frac{\partial I(\mathbf{x},t)}{\partial t} = \Delta I(\mathbf{x},t),$$

$$I^{\text{new}}(\mathbf{x}) = \int_{\Omega} g_{\sigma}(|\mathbf{x}-\mathbf{y}|) I(\mathbf{y}) d\mathbf{y},$$



- Anisotropic (Nonlinear) Diffusion: P. Perona and J. Malik, IEEE PAMI, 1990.

$$\frac{\partial I(\mathbf{x},t)}{\partial t} = div(g_{\sigma}(|\nabla I(\mathbf{x},t)|^2)\nabla I(\mathbf{x},t)), \quad g_a(r) = \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2a^2}}$$

- Total Variation: L. Rudin, S. Osher, and E. Fatemi, Physica D, 1992.

$$\arg\min_{I^{new}} \lambda \int_{\Omega} \phi(|\nabla I^{new}(\mathbf{x})|) d\mathbf{x} + \frac{1}{2}\int_{\Omega} |g_{\sigma} * I^{new}(\mathbf{x}) - I(\mathbf{x})|^2 \, d\mathbf{x}$$

$$\Rightarrow \lambda \, \text{div}\left(\frac{\phi'(|\nabla I^{new}(\mathbf{x})|)}{|\nabla I^{new}(\mathbf{x})|} \nabla I^{new}(\mathbf{x})\right) = (g_{\sigma} * I^{new}(\mathbf{x}) - I(\mathbf{x})) * g_{\sigma}(-\mathbf{x})$$

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Bilateralフィルタ

- **A natural extension of the Gaussian filter:**

$$I^{\text{new}}(\mathbf{x}) = \frac{\int \exp(-|\mathbf{x}-\mathbf{y}|^2/\sigma^2)\exp(-|I(\mathbf{x})-I(\mathbf{y})|^2/h^2) I(\mathbf{y}) d\mathbf{y}}{\int \exp(-|\mathbf{x}-\mathbf{y}|^2/\sigma^2)\exp(-|I(\mathbf{x})-I(\mathbf{y})|^2/h^2) d\mathbf{y}}$$

Input image intensity: $I(\mathbf{x})$ Pixel/voxel position: $\mathbf{x},\mathbf{y}$

Output filtered intensity: $I^{\text{new}}(\mathbf{x})$ User-specified parameters: σ, h

- Nonlinear Gaussian Filter: V. Aurich and J. Weule, 1995.
- SUSAN Filter: S. Smith and J. Brady, 1997.
- Bilateral Filter: C. Tomasi and R. Manduchi, 1998.

- **Popular: very simple form but powerful ability:**
  エッジ特徴を保存する！
- **Many applications in Computational Photography.**

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Bilateralフィルタとは？

Gaussian Filter ⇐ Input ⇒ Bilateral Filter

$$Z(\mathbf{x},\mathbf{y}) = g_{\sigma}(|\mathbf{x}-\mathbf{y}|) \qquad Z(\mathbf{x},\mathbf{y}) = g_h(|I(\mathbf{x})-I(\mathbf{y})|) g_{\sigma}(|\mathbf{x}-\mathbf{y}|)$$

$$g_a(r) = e^{-\frac{r^2}{a^2}}.$$

Intensity (Tonal) Kernel — Spatial Kernel

Spatial-Tonal Normalized Convolution:

$$I^{\text{new}}(\mathbf{x}) = \int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y} \Big/ \int Z(\mathbf{x},\mathbf{y}) d\mathbf{y},$$

エッジ特徴を保存する！

- Many extensions…
- Non-Local Means Filter: A. Buades, B. Coll, and J.-M. Morel, 2004.

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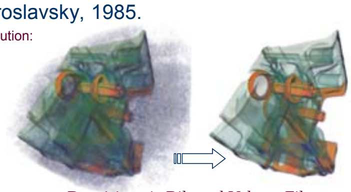
Neighborhood Filtering

- Yaroslavsky Filter: L. Yaroslavsky, 1985.
  - Spatial-Tonal Normalized Convolution:

$$I^{\text{new}}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y}}{\int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}},$$

Convolution Kernel:

$$Z(\mathbf{x},\mathbf{y}) = g_h(|I(\mathbf{x})-I(\mathbf{y})|),$$



Denoising via Bilateral Volume Filter

- Nonlinear Gaussian Filter: V. Aurich and J. Weule, 1995.
- SUSAN Filter: S. Smith and J. Brady, 1997.
- Bilateral Filter: C. Tomasi and R. Manduchi, 1998.

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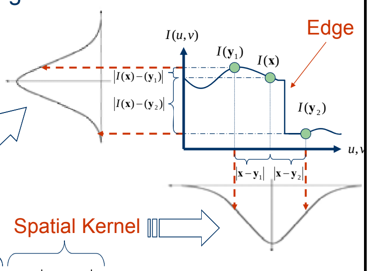
なぜエッジを保存するのか？

- Intensity Kernel: Suppresses averaging across the edge.
- Spatial Kernel: Localizes the suppression.

$$I^{\text{new}}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y}}{\int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}}, \quad g_a(r) = e^{-\frac{r^2}{a^2}}.$$



Intensity Kernel — Spatial Kernel

$$Z(\mathbf{x},\mathbf{y}) = g_h(|I(\mathbf{x})-I(\mathbf{y})|) g_{\sigma}(|\mathbf{x}-\mathbf{y}|)$$

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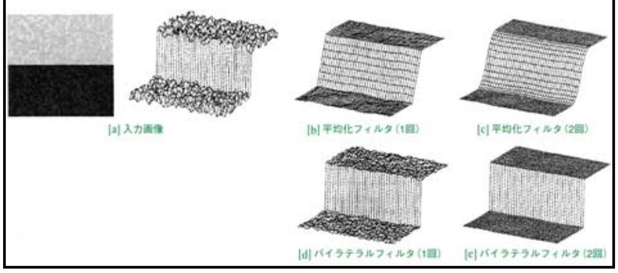
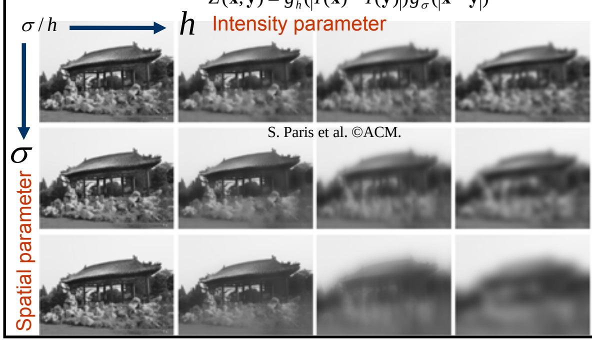
Bilateralフィルタ



Parameters of Bilateral Filtering

- From S. Paris, et al. *A gentle introduction to bilateral filtering and its applications,* In ACM SIGGRAPH '07 courses, IEEE CVPR'08 Tutorials, and ACM SIGGRAPH Asia'08 classes.

$$Z(\mathbf{x},\mathbf{y}) = g_h(|I(\mathbf{x}) - I(\mathbf{y})|)g_\sigma(|\mathbf{x}-\mathbf{y}|)$$



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研究・応用:
高速Bilateralフィルタ

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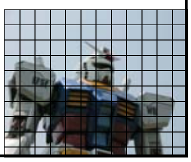
高速近似法研究の背景 & Motivation

✓ Non-local(非局所的な積分が好ましい): 画像全体の重み付平均が各画素に対して必要。重みは各画素で異なる。

✓ 計算量が非常に多い (Non-Linear): 高速化何もなしの exact bilateralフィルタは$O(N^2)$計算複雑度(quadratic computational complexity)。ここでNは画素数.

✓莫大な計算時間が必要!
→ 30 years 512^3 voxels by 3.2 GHz.

Bilateral Filter: $I^{new}(\mathbf{x}) = \int Z(\mathbf{x},\mathbf{y})I(\mathbf{y})d\mathbf{y} / \int Z(\mathbf{x},\mathbf{y})d\mathbf{y},$



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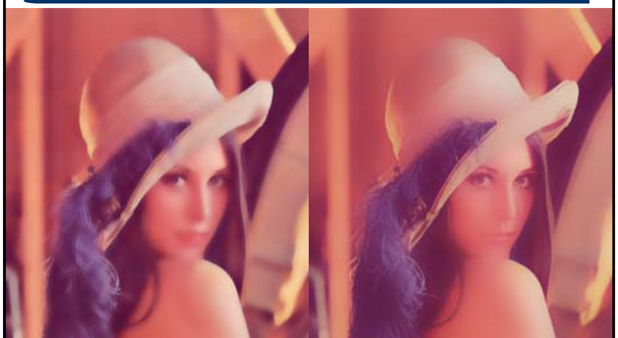
局所領域で平均化→アーティファクト(リング)



Local 15x15 Global

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既存の高速Bilateralフィルタ近似法群

✓ Exact bilateral filtering requires $O(N^2)$計算複雑度.
✓ There are fast approximations (高速近似法):

- **Separable Filtering Approach $O(N^{3/2})$:**
  - T. Pham and L. Vliet, IEEE ICME, 2005.
- **FFT-based Approaches O(N log N):**
  - F. Durand and J. Dorsey, ACM SIGGRAPH, 2002. WEBソースコードあり
  - J. Chen, J. Paris, and F. Durand, ACM SIGGRAPH, 2007.
  - S. Paris and F. Durand, ECCV, 2006, IJCV, 81(1), 2009.
- **Histogram-based Approaches O(N):**
  - B. Weiss, ACM SIGGRAPH, 2006.
  - F. Porikli, IEEE CVPR, 2008. WEBソースコードあり
- **Kd-Tree-based Approach O(N log N):** WEBソースコードあり
  - A. Adams, N. Gelfand, J. Dolson, and M. Levoy, ACM SIGGRAPH, 2009.
- **FGT-based Approach O(N):**
  - S. Yoshizawa, A. Belyaev, and H. Yokota, CGF, 2010.

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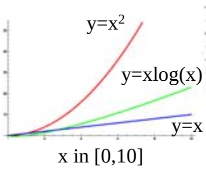
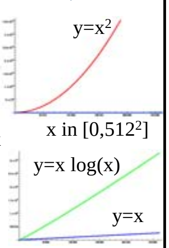
Computational Complexity

✓ Big O notation proposed by P. Bachmann, 1892.
 -Required number of operations w.r.t. some number.
 -Best, worst, and average cases: worst-analysis is common.
✓ O(1): constant.
Polynomial Complexities:
✓ O(N): Linear.
✓ O(N log N): Sub-linear.
✓ $O(N^2)$: Quadratic.
✓...
Exponential Complexity:
✓ $O(k^N)$: for some k.





$O(N^2) \rightarrow O(N)$: 512^3 times faster if N= 512^3.

✓ In practice, linear is very fast, sub-linear is fast, and quadratic possibly work as 31 years 512^3 voxels by 3.2 GHz PC.
$O(N^2) \rightarrow O(230N)$: if the linear algorithm takes 3 min for 512^3.

Discrete Bilateral Filter for Images & Volumes

- ✓ **Smooth continuous bilateral filter:**
  - Spatial-Tonal Normalized Convolution:

$$I^{new}(\mathbf{x}) = \int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y} / \int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}, \quad Z(\mathbf{x},\mathbf{y}) = g_h(|I(\mathbf{x}) - I(\mathbf{y})|) g_\sigma(|\mathbf{x}-\mathbf{y}|)$$

- ✓ **Discrete bilateral filter: usually the following form is called the bilateral filter in image processing.**

2D Case (Image):

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})|) g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}),$$

$$W_{l,m} = \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})|) g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|), \quad s_x s_y = N.$$

3D Case (Volume):

$$I^{new}_{l,m,n} = \frac{1}{W_{l,m,n}} \sum_{i=1}^{s_z} \sum_{j=1}^{s_y} \sum_{k=1}^{s_x} g_h(|I(\mathbf{x}_{i,j,k}) - I(\mathbf{x}_{l,m,n})|) g_\sigma(|\mathbf{x}_{i,j,k} - \mathbf{x}_{l,m,n}|) I(\mathbf{x}_{i,j,k}),$$

$$W_{l,m,n} = \sum_{i=1}^{s_z} \sum_{j=1}^{s_y} \sum_{k=1}^{s_x} g_h(|I(\mathbf{x}_{i,j,k}) - I(\mathbf{x}_{l,m,n})|) g_\sigma(|\mathbf{x}_{i,j,k} - \mathbf{x}_{l,m,n}|), \quad s_x s_y s_z = N.$$

Separable Filtering Approach

T. Pham and L. Vliet, IEEE ICME, 2005.

- ✓ $O(N^{3/2})$: quasi-linear computational complexity.
- ✓ Regular spatial sampled data.
- ✓ HDR without heuristics.
- ✓ High-dimensional data: applicable but poor quality.
- ✓ Quaint fast and low memory cost but poor quality.

Consider the Gaussian: $g_a(r) = e^{-\frac{r^2}{a^2}}.$

$$\mathbf{x}_{i,j} = (i,j), \boldsymbol{\sigma} = (\sigma_x, \sigma_y) \Rightarrow g_{\boldsymbol{\sigma}}(|\mathbf{x}_{i,j}|) = \exp(-(\frac{j^2}{\sigma_x^2} + \frac{i^2}{\sigma_y^2})).$$

$$g_{\boldsymbol{\sigma}}(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) = \exp(-(\frac{(j-m)^2}{\sigma_x^2} + \frac{(i-l)^2}{\sigma_y^2})) = g_{\sigma_x}(|j-m|) g_{\sigma_y}(|i-l|).$$

Gaussian Filter: $I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}),$

Separable Filtering Approach

T. Pham and L. Vliet, IEEE ICME, 2005.

$$g_{\boldsymbol{\sigma}}(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) = \exp(-(\frac{(j-m)^2}{\sigma_x^2} + \frac{(i-l)^2}{\sigma_y^2})) = g_{\sigma_x}(|j-m|) g_{\sigma_y}(|i-l|).$$

Gaussian Filter:

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}) = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} g_{\sigma_y}(|i-l|) (\sum_{j=1}^{s_x} g_{\sigma_x}(|j-m|) I(\mathbf{x}_{i,j})).$$

Separable Gaussian Filter:

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} g_{\sigma_y}(|i-l|) J_{i,m}, \quad J_{i,m} = \sum_{j=1}^{s_x} g_{\sigma_x}(|j-m|) I(\mathbf{x}_{i,j}).$$

Separable Bilateral Filter:

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} g_h(|I(\mathbf{x}_{i,m}) - I(\mathbf{x}_{l,m})|) g_{\sigma_y}(|i-l|) J_{i,m}, \quad J_{i,m} = \sum_{j=1}^{s_x} g_{\sigma_x}(|j-m|) g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{i,m})|) I(\mathbf{x}_{i,j}).$$

Assume $s_x = s_y = sqrt(N)$. For each i column, $J_{i,m}$ is calculated by, i.e. sqrt(N)*sqrt(N) for each i. There is sqrt(N) columns, then $O(N^{3/2})$. $I_{l,m}$ is also calculated by $O(N^{3/2})$.

FFT-based Approaches

F. Durand and J. Dorsey, ACM SIGGRAPH, 2002, J. Chen, J. Paris, and F. Durand, ACM SIGGRAPH, 2007, & S. Paris and F. Durand, ECCV, 2006, IJCV, 81(1), 2009.

- ✓ O(N log N): sub-linear computational complexity.
- ✓ Regular spatial sampled data.
- ✓ HDR with resampling & linear interpolations.
- ✓ High-dimensional data: applicable but high memory cost.
- ✓ Fast and high memory cost, and quality control (resampling).

Consider the Gaussian Filter:

$$I^{new}(\mathbf{x}) = \int g_\sigma(|\mathbf{x}-\mathbf{y}|) I(\mathbf{y}) d\mathbf{y} / \int g_\sigma(|\mathbf{x}-\mathbf{y}|) d\mathbf{y} = \frac{F^{-1}[F[g]F[I]]}{F^{-1}[F[g]F[P]]}, \; P(\mathbf{x}) = 1.$$

2D FFT-based Bilateral Filter[DD02]:

$$J(\mathbf{x}_{l,m}, \eta) = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_h(|I(\mathbf{x}_{i,j}) - \eta|) g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}), \quad I^{new}_{l,m} = J(\mathbf{x}_{l,m}, I(\mathbf{x}_{l,m})).$$

FFT-based Approaches

F. Durand and J. Dorsey, ACM SIGGRAPH, 2002, J. Chen, J. Paris, and F. Durand, ACM SIGGRAPH, 2007, & S. Paris and F. Durand, ECCV, 2006, IJCV, 81(1), 2009.

$$J(\mathbf{x}_{l,m}, \eta) = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_h(|I(\mathbf{x}_{i,j}) - \eta|) g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}), \quad I^{new}_{l,m} = J(\mathbf{x}_{l,m}, I(\mathbf{x}_{l,m})).$$

Construct M 2D images where the image intensity space is resampled by M. Denote each sample by eta. Apply M 2D discrete FFT to J(), it gives us 6M 2D FFTs: O(12MN log N). Filtered image I^{new} is obtained by linearly interpolating J() at $I(x_{l,m})$ in intensity space.

3D FFT-based Bilateral Filter[PD06,PD09]:

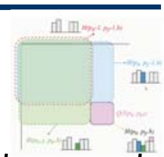
$$I^{new}(\mathbf{x}) = J(\mathbf{x}, I(\mathbf{x})) = \int g_\sigma(|\mathbf{u}-\mathbf{v}|) f(\mathbf{v}) d\mathbf{v} / \int g_\sigma(|\mathbf{u}-\mathbf{v}|) d\mathbf{v} = \frac{F^{-1}[F[g]F[f]]}{F^{-1}[F[g]F[P]]},$$

$$\mathbf{u} = (\mathbf{x}, u), \mathbf{v} = (\mathbf{y}, v), \; f(\mathbf{v}) = \delta(v - I(\mathbf{y})) I(\mathbf{y}), P(\mathbf{v}) = 1.$$

→Bilateral Filtering as Convolution

Histogram-based Approaches

B. Weiss, ACM SIGGRAPH, 2006 & F. Porikli, IEEE CVPR, 2008.



- ✓ O(N): linear computational complexity.
- ✓ Regular spatial sampled data.
- ✓ Heuristic for non-box spatial kernels.
- ✓ HDR with complex heuristic procedures.
- ✓ High-dimensional data: applicable but high memory cost.
- ✓ Fast and high memory cost, quality control (bins).

Discrete Bilateral Filter: $I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{s_y} \sum_{j=1}^{s_x} g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})|) g_\sigma(|\mathbf{x}_{i,j} - \mathbf{x}_{l,m}|) I(\mathbf{x}_{i,j}),$

Box-Spatial Bilateral Filter: $I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=l-h_y}^{l+h_y} \sum_{j=m-h_x}^{m+h_x} g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})|) I(\mathbf{x}_{i,j}),$

Assume that intensity I() live in the natural number space. Consider the M sequential of natural number and denote it by Omega={eta_1, eta_2, …eta_M}. Then,

$$I(\mathbf{x}_{i,j}), |I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})| \in \Omega = \{\eta_1, \eta_2, ..., \eta_M\}.$$

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Histogram-based Approaches

B. Weiss, ACM SIGGRAPH, 2006 & F. Porikli, IEEE CVPR, 2008:

$$I(\mathbf{x}_{i,j}), |I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})| \in \Omega = \{\eta_1, \eta_2, \ldots, \eta_M\}.$$

Box-Spatial Bilateral Filter: $I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=l-h_y}^{l+h_y} \sum_{j=m-h_x}^{m+h_x} g_h(|I(\mathbf{x}_{i,j}) - I(\mathbf{x}_{l,m})|) I(\mathbf{x}_{i,j}),$

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} (g_h(|\eta_1 - I(\mathbf{x}_{l,m})|)\#(\eta_1)\eta_1 + g_h(|\eta_2 - I(\mathbf{x}_{l,m})|)\#(\eta_2)\eta_2 + \ldots + g_h(|\eta_M - I(\mathbf{x}_{l,m})|)\#(\eta_M)\eta_M),$$

$$I^{new}_{l,m} = \frac{1}{W_{l,m}} \sum_{i=1}^{M} g_h(|i - I(\mathbf{x}_{l,m})|)\mathrm{bin}(i)\eta_i. \quad (2h_x+1)(2h_y+2) = M.$$

Since it is equivalent to calculate I^{new} and counting same eta and sum up all eta within box, this filtering algorithm problem is converted to construct histogram of I(). The computational complexity is linear w. r. t. the elements because the number of bins is constant.

Fast 3D Histogram Construction Algorithms:
- ✓ B. Weiss, ACM SIGGRAPH, 2006, F. Porikli, IEEE CVPR, 2008, CVPR 2007.

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Kd-Tree-based Approach

A. Adams, N. Gelfand, J. Dolson, and M. Levoy, ACM SIGGRAPH, 2009.
- ✓ O(N log N): sub-linear computational complexity.
- ✓ Non-uniformly sampled data.
- ✓ HDR without heuristics.
- ✓ Gaussian approximation is heuristic truncations.
- ✓ High-dimensional data: applicable with low memory cost.
- ✓ Fast and low memory cost but quality control (tree level).

Similar to FFT-based approaches [PD06,PD09],
→Bilateral Filtering as Convolution.
Then, apply Kd-tree approximations of Gaussian function. Constructing and evaluating Kd-tree both require O(N log N).

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我々の新しい計算法: 高速Bilateralガウス変換

Exact: 非線形 & O(N²) Cost:

$$I^{new}(x) = \frac{\int g_1(x-y) g_2(I(x)-I(y)) I(y) dy}{\int g_1(x-y) g_2(I(x)-I(y)) dy}, \quad y \in \mathbb{R}^n$$ nD

Bilateral Filtering as Convolution
✓空間拡張:Dimension Elevation Technique:

$$J(x,u) = \frac{\int_{\mathbb{R}}\int_{\mathbb{R}^n} f(y,v) g_1(x-y) g_2(u-v) dy dv}{\int_{\mathbb{R}}\int_{\mathbb{R}^n} h(y,v) g_1(x-y) g_2(u-v) dy dv},$$ (n+1)D

where $f(y,v) = h(y,v)I(y)$, $h(y,v) = \delta(v - I(y))$, $x, y \in \mathbb{R}^n$

高速ガウス変換: 線形 & O(N) Cost:

$$I^{new}(x_i) = \frac{\sum_{j=1}^{M} q_j G(\mathbf{u}_i - \mathbf{v}_j)}{\sum_{j=1}^{M} G(\mathbf{u}_i - \mathbf{v}_j)} = \frac{\mathrm{GT}(\mathbf{q}, \mathbf{u}_i, \mathbf{v}, M)}{\mathrm{GT}(1, \mathbf{u}_i, \mathbf{v}, M)}$$ ⇦ FGT

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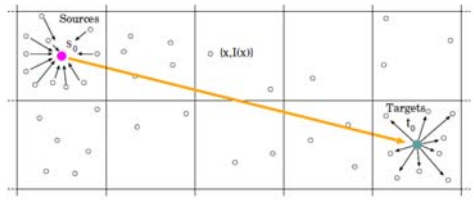
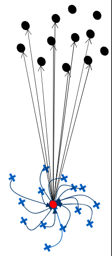
高速ガウス変換: Fast Gauss Transform (FGT)





✓Main Idea: near-fieldとfar-fieldでのガウス関数の漸近展開による迅速な評価:

$$\frac{\sum_{j=1}^{M} q_j G(\mathbf{u}_i - \mathbf{v}_j)}{\sum_{j=1}^{M} G(\mathbf{u}_i - \mathbf{v}_j)}$$ G(t-s)

✓O(N) 計算複雑度かつ精度の制御が解析的に可能!

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1次元高速ガウス変換Overview

- far-fieldの source点群をHermite展開

$$\sum_{k=0}^{\infty} [\frac{1}{k!}(\frac{s-s_0}{\sigma})^k] h_k(\frac{t-s_0}{\sigma}) = \sum_{k=0}^{\infty} A_k h_k(\frac{t-s_0}{\sigma}),$$

where $h_k(x) = (-1)^k \frac{d^k}{dx^k}\exp(-x^2)$ are Hermite functions

- Sourceとtargetを交換しnear-filed でtargetをTaylor級数で表現

$$G(t-s) = \sum_{l=0}^{\infty} [\frac{1}{l!} h_l(\frac{s-t_0}{\sigma})](\frac{t-t_0}{\sigma})^l = \sum_{l=0}^{\infty} B_l (\frac{t-t_0}{\sigma})^l,$$

$$B_l = \frac{(-1)^l}{l!} \sum_{k=0}^{\infty} A_k h_{k+l}(\frac{s_0-t_0}{\sigma}).$$

- ガウス関数は急速に零になるので、限られた項数(p)だけの和で近似可能!

Algorithm 1 Fast Gauss Transform

Subroutine: FGT($n, \sigma, \{\alpha\}, \{t\}, \{s\}, N, M, \epsilon$)
Inputs: error ϵ, dimension n, bandwidth parameters $\sigma \in \mathbb{R}^n$, targets $\{t\}$, sources $\{s\}$, number of targets N, number of sources M, and scalar weights $\{\alpha\}$.
Inside Parameters: box length w and interaction region radius r.
Outputs: a scalar set $\{\beta\}$.
Require: $\epsilon > 0$ and $\alpha_j \in \{\alpha\} \geq 0$.

```
Set up box partitions with side length w(σ) for s and t.
Compute the number of kept terms p from ε via an error
  estimator (we use that derived in [WK06]).
for all source boxes do
  for k = 0 to p do
    Compute the Hermite expansion coefficients A_k.
  end for
end for
for all target boxes i do
  for all interaction region within (2r+1)^n nearest boxes do
    for k = 0 to p do
      Compute the Taylor expansion coefficients B_l with A_k.
    end for
  end for
end for
for all sources do
  Evaluate the Taylor expansion.
  for k = 0 to p do
    Summing up the Taylor series with B_k.
  end for
  Store the sum to a scalar set {β}.
end for
Return {β}.
```

nD: Multi-Index Notations

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高速ガウス変換の誤差評価

- 与えられた誤差パラメータ(epsilon) を満たす漸近展開の項数(p)を事前に計算可能か?

Yes we can !

- 与えられたepsilon とfar/near- fieldの相互作用範囲の半径(r)を用いて,項数 p はFGT error estimatesによって解析的に計算が可能!

- FGT error estimates:
  - ✓ L. Greengard and J. Strain, SIAM J. of SSC, 1991.
  - ✓ L. Greengard and X. Sun, IEEE Int. Conf. on Mathematics III, 1998.
  - ✓ B. Baxter and G. Roussos, SIAM J. of SSC, 2002.
  - ✓ X. Wan and G. E. Karniadakis, J. of Computational Physics, 2006.

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我々の新しいO(N) Bilateral Filtering Algorithm

1. 精度とbilateralフィルタのパラメータを与える:
   - r: radius of far/near-field interactions:
     - r=4(float)
     - r=6(double)
   - epsilon: a user specified error parameter.
2. Dataを(n+1)次元へ.
3. FGTs を適用:
   - Sharp Error Estimate: X. Wan and G. E. Karniadakis, J. of Computational Physics, 2006.

Algorithm 2 $O(N)$ Bilateral Filter

Input: error ε, dimensionality n, spatial bandwidths $\sigma^{g_1} \in \mathbb{R}^n$, tonal bandwidth $\sigma^{g_2} \in \mathbb{R}$, targets $\{\mathbf{x}\}$, sources $\{\mathbf{y}\}$, number of targets N, number of sources M, scalar image intensity $\{I(\mathbf{x})\}$.
Called Functions: FGT(·).
Output: $\{I^{\text{new}}(\mathbf{x})\}$.
Require: $E > 0, \exists I(\mathbf{x}) \geq 0, \exists I(\mathbf{y}) \geq 0$.
1: $\{q\} \leftarrow \{I(\mathbf{y}_1), I(\mathbf{y}_2), ..., I(\mathbf{y}_M)\}, \mathbf{y}_j \in \mathbf{y}$.
2: $\{1\} \leftarrow \{1, 1, ...1\}$.
3: $\{\mathbf{u}\} \leftarrow \{\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_N\} : \mathbf{u} \ni \mathbf{u}_i \leftarrow (\mathbf{x}_i, I(\mathbf{x}_i))$.
4: $\{\mathbf{v}\} \leftarrow \{\mathbf{v}_1, \mathbf{v}_2, ..., \mathbf{v}_M\} : \mathbf{v} \ni \mathbf{v}_j \leftarrow (\mathbf{y}_j, I(\mathbf{y}_j))$.
5: $\sigma \leftarrow (\sigma^{g_1}, \sigma^{g_2})$.
6: $\{f_i\} \leftarrow \text{FGT}((n+1), \sigma, \{q\}, \{\mathbf{u}\}, \{\mathbf{v}\}, N, M, E)$.
7: $\{g_i\} \leftarrow \text{FGT}((n+1), \sigma, \{1\}, \{\mathbf{u}\}, \{\mathbf{v}\}, N, M, E)$.
8: **for** $i = 1$ to N **do**
9: $\quad I^{\text{new}}(\mathbf{x}_i) \leftarrow \frac{f_i}{g_i}$.
10: **end for**
11: **Return** $\{I^{\text{new}}(\mathbf{x})\} \leftarrow \{I^{\text{new}}(\mathbf{x}_1), I^{\text{new}}(\mathbf{x}_2), ... I^{\text{new}}(\mathbf{x}_N)\}$.

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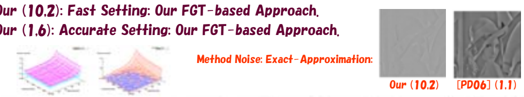
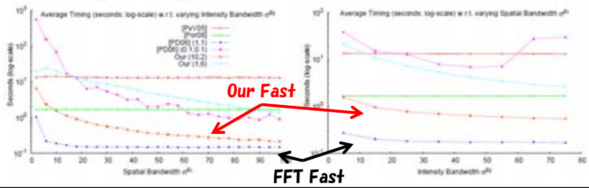
数値計算実験: 速度比較

- Comparison with conventional fast bilateral filters:
  - [PvV05]: Separable Filtering: T. Pham and L. Vliet, IEEE ICME, 2005.
  - [Por08]: Accurate Setting: Histogram-based: F. Porikli, IEEE CVPR, 2008.
  - [PD06] (1,1): Fast Setting: FFT-based: S. Paris and F. Durand, ECCV, 2006.
  - [PD06] (0.1,0.1): Accurate Setting: FFT-based: S. Paris and F. Durand, ECCV, 2006.
  - Our (10,2): Fast Setting: Our FGT-based Approach.
  - Our (1,6): Accurate Setting: Our FGT-based Approach.





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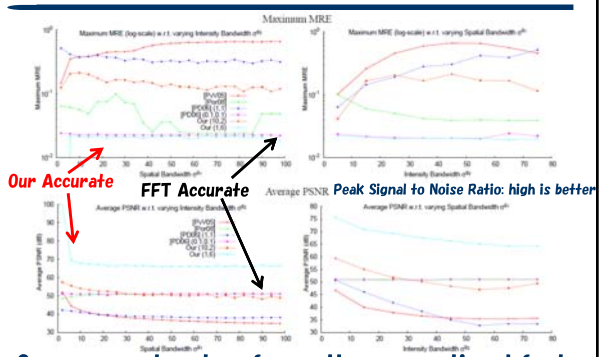
数値計算実験: 精度比較



- Our approach outperforms the conventional fast bilateral filters in terms of speed and accuracy !

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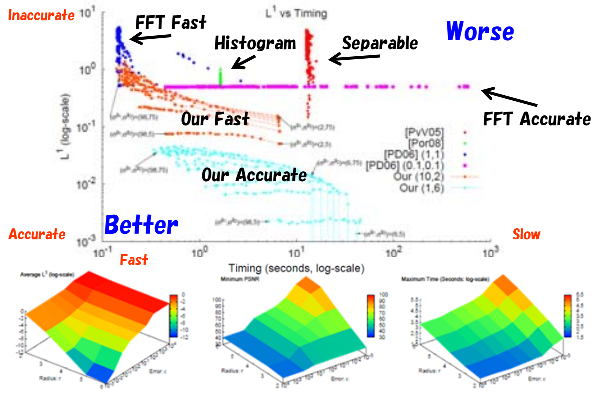
数値計算実験: 精密制御性(Precision Control)の比較



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Comparison with Supercomputers & Specialized Hardwares

- Gauss transform consists of sums & multiplications.
- N-body problem, kernel density estimation: molecular dynamics, galaxy collisions, etc.
- Why not to use the specialized hardwares (e.g. MDGrape cluster: 3 Tera FLOPS =96.0 Giga FLOPS/node×32nodes=256cores) or supercomputers (e.g. IBM Roadrunner 1.1 Peta FLOPS: 129600 cores, Cray Jaguar 106Tera FLOPS: 150152 cores, top 500 ranking, 2009, June) ?
- Usual 3.3 GHz (Core i7) PC approximately has 51 Giga FLOPS.
  - Specialized hardware: 100 times faster but O(N²/100).

MDGrape3 cluster RIKEN.

512^3 via 30 years -> 113 days.

  - Supercomputer: 2x10⁴ times faster but O(N²/2x10⁴).

RIKEN Super Combined Cluster. System

512^3 via 30 years -> 13.6 hours.

Isn't this good ?

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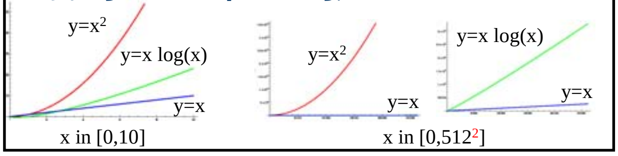
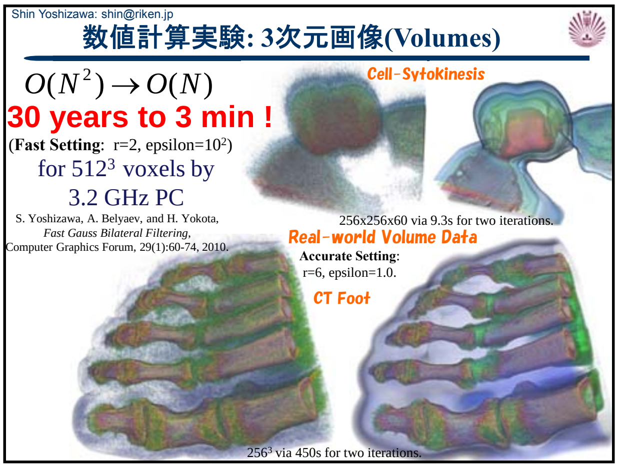
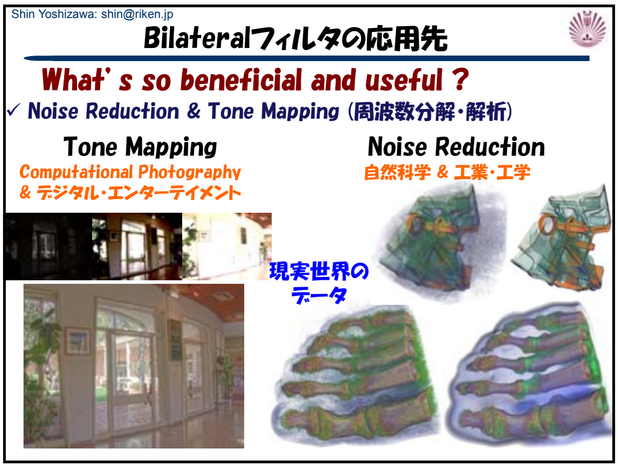
Comparison with Supercomputers & Specialized Hardwares

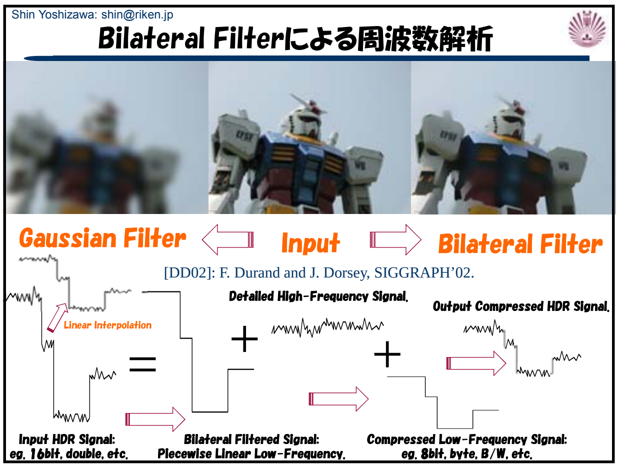
No !, a quadratic is a quadratic is a quadratic.

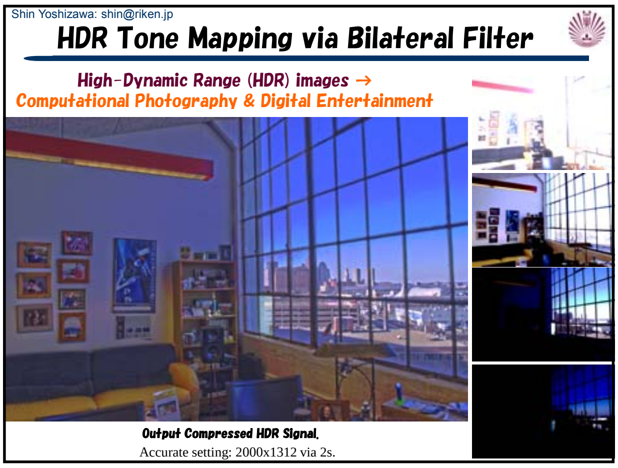
- Naive: 512^3->1024^3: 30 years-> 30x64 years.
- Specialized hardware: 512^3->1024^3: 113x64 days-> 20 years.
- Supercomputer: 512^3->1024^3: 13.6x64 hours-> 36 days.
- Our O(N) algorithm: 512^3->1024^3: 3x8min -> 24 min !!!
- Reducing complexity v.s. use of hardwares: Out of the question, although hardware acceleration of O(N) algorithm is promising.

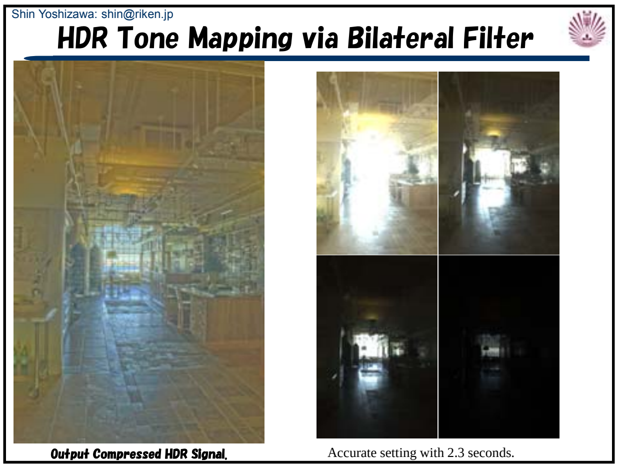


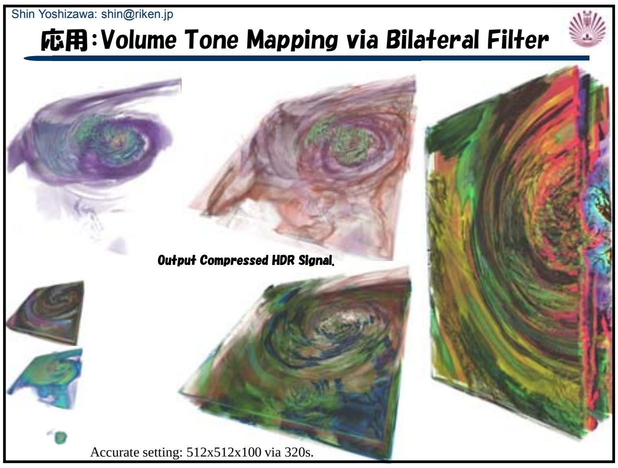

Shin Yoshizawa: shin@riken.jp
数値計算実験: 3次元画像(Volumes)
$O(N^2) \rightarrow O(N)$
30 years to 3 min !
(Fast Setting: r=2, epsilon=10²)
for 512³ voxels by
3.2 GHz PC
S. Yoshizawa, A. Belyaev, and H. Yokota,
Fast Gauss Bilateral Filtering,
Computer Graphics Forum, 29(1):60-74, 2010.
Cell-Sytokinesis
256x256x60 via 9.3s for two iterations.
Real-world Volume Data
Accurate Setting:
r=6, epsilon=1.0.
CT Foot
256³ via 450s for two iterations.


Shin Yoshizawa: shin@riken.jp
Bilateralフィルタの応用先
What's so beneficial and useful ?
✓ Noise Reduction & Tone Mapping (周波数分解・解析)
Tone Mapping
Computational Photography
& デジタル・エンターテイメント
Noise Reduction
自然科学 & 工業・工学
現実世界の
データ


Shin Yoshizawa: shin@riken.jp
Bilateral Filterによる周波数解析
Gaussian Filter
Input
Bilateral Filter
[DD02]: F. Durand and J. Dorsey, SIGGRAPH'02.
Detailed High-Frequency Signal.
Output Compressed HDR Signal.
Linear Interpolation
Input HDR Signal:
eg. 16bit, double, etc.
Bilateral Filtered Signal:
Piecewise Linear Low-Frequency.
Compressed Low-Frequency Signal:
eg. 8bit, byte, B/W, etc.


Shin Yoshizawa: shin@riken.jp
HDR Tone Mapping via Bilateral Filter
High-Dynamic Range (HDR) images →
Computational Photography & Digital Entertainment
Output Compressed HDR Signal.
Accurate setting: 2000x1312 via 2s.


Shin Yoshizawa: shin@riken.jp
HDR Tone Mapping via Bilateral Filter
Output Compressed HDR Signal.
Accurate setting with 2.3 seconds.


Shin Yoshizawa: shin@riken.jp
応用:Volume Tone Mapping via Bilateral Filter
Output Compressed HDR Signal.
Accurate setting: 512x512x100 via 320s.

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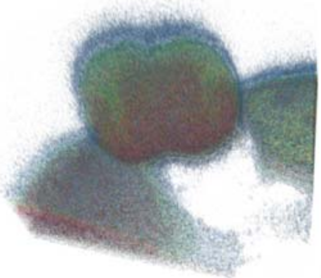
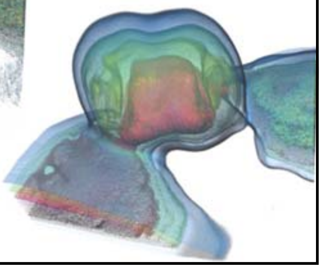
Possible Tone Mapping Application in Cell Biology



✓ Different exposures, laser powers, or etc. provides HDR cell images.



✓ HDR cell images may reveal hidden biological objects & functions.

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最新のBilateral-likeフィルタ

- Screened Poisson Equation:
  - ✓ M. Chuang and M. Kazhdan. ACM SIGGRAPH 2011.

- Domain Transform:
  - ✓ E. Gastal and M. Oliveira. ACM SIGGRAPH. 2011.

WEBプログラムあり

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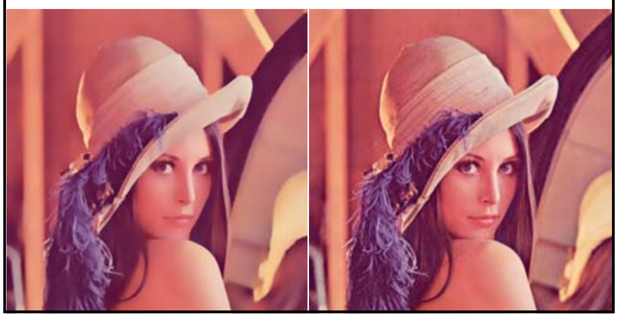
Domain Transform



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ノイズ除去

パターン保存フィルタ
—平滑化(Smoothing)—

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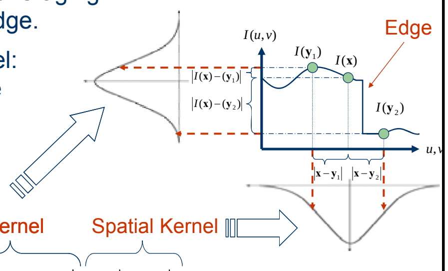
復習：エッジ保存

✓ Intensity Kernel: Suppresses averaging across the edge.

✓ Spatial Kernel: Localizes the suppression.

$$I^{\text{new}}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y}}{\int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}}, \quad g_a(r) = e^{-\frac{r^2}{a^2}}.$$



Intensity Kernel　Spatial Kernel

$$Z(\mathbf{x},\mathbf{y}) = \underbrace{g_h(|I(\mathbf{x})-I(\mathbf{y})|)}_{\text{Intensity Kernel}}\underbrace{g_\sigma(|\mathbf{x}-\mathbf{y}|)}_{\text{Spatial Kernel}}$$

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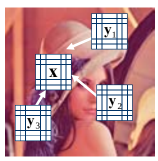
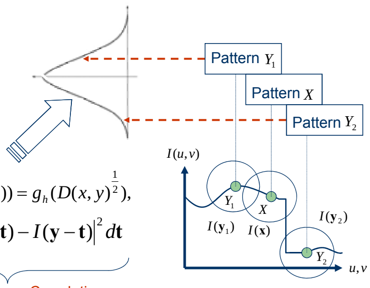
Pattern Preserving Filter

- Non-Local (NL-) Means Filter: A. Buades, B. Coll, and J.-M. Morel, 2004~.

✓ Similarity Kernel: Suppresses averaging across the pattern.

$$I^{\text{new}}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y}}{\int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}}, \quad g_a(r) = e^{-\frac{r^2}{a^2}}.$$





Similarity Kernel

$$Z(\mathbf{x},\mathbf{y}) = g_h(\text{Distance}(X,Y)) = g_h(D(x,y)^{\frac{1}{2}}),$$

$$D(\mathbf{x},\mathbf{y}) = \underbrace{\int_{\Re^l} g_\sigma(\mathbf{t})|I(\mathbf{x}-\mathbf{t}) - I(\mathbf{y}-\mathbf{t})|^2 d\mathbf{t}}_{\text{Gaussian Cross-Correlation}}$$

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Non-Local Mean Image Filtering

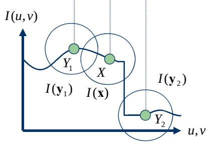
- Antoni Buades, Bartomeu Coll, and Jean-Michel Morel, 2004~.
- ✓ **Video Processing**: Bennett and McMillan Sig'05, Mahmoudi and Sapiro SPL'05.

Denoising via NL-Mean

$$I^{new}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y}}{\int Z(\mathbf{x},\mathbf{y}) d\mathbf{y}}, \quad g_\sigma(r) = e^{-\frac{r^2}{\sigma^2}}.$$

Similarity Kernel:

$$Z(\mathbf{x},\mathbf{y}) = g_h(\mathrm{Distance}(X,Y)) = g_h(D(x,y)^{\frac{1}{2}}),$$



$$D(\mathbf{x},\mathbf{y}) = \int_{\Re^2} g_\sigma(\mathbf{t}) \left| I(\mathbf{x}-\mathbf{t}) - I(\mathbf{y}-\mathbf{t}) \right|^2 dt$$

Gaussian Cross-Correlation

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Similarity Measure: Bilateral vs. NL-Mean

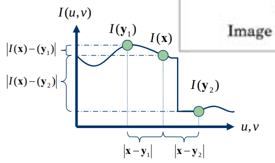
Bilateral Filter

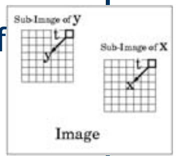
$$Z(\mathbf{x},\mathbf{y}) = g_h(D(\mathbf{x},\mathbf{y})) g_\sigma(|\mathbf{x}-\mathbf{y}|),$$
$$D(\mathbf{x},\mathbf{y}) = |I(\mathbf{x}) - I(\mathbf{y})|$$

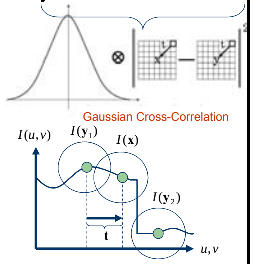
Non-Local Mean

$$Z(\mathbf{x},\mathbf{y}) = g_h(D(\mathbf{x},\mathbf{y})^{\frac{1}{2}}),$$
$$D(\mathbf{x},\mathbf{y}) = \int g_\sigma(\mathbf{t}) \left| I(\mathbf{x}-\mathbf{t}) - I(\mathbf{y}-\mathbf{t}) \right|^2 dt$$

Difference of a pixel pair.







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Computational Complexity O(W²N²)

- ✓ Averaging fixed size of the neighborhood region.

Buades et al. 7x7 window with summing up 21x21.

- ✓ Averaging subset of image. Statistical Analysis for Gradient: M. Mahmoudi and G. Sapiro,
- **Kd-Tree-based Approach O(N log N):**
  - ✓ **A. Adams, N. Gelfand, J. Dolson, and M. Levoy, ACM SIGGRAPH, 2009.**

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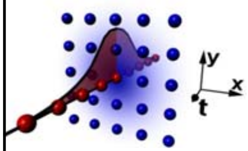
NL-Mean Filter to Video (3D Image)

- ✓ Video Enhancement Using Pre-Pixel Virtual Exposures
  - Eric P. Bennett and Leonard McMillan, SIGGRAPH 2005.
  - ASTA: Adaptive Spatio-Temporal Accumulation Filter.

Tone Mapping + Temporal NL-Mean Filter (ASTA)

$$I^{new}(\mathbf{x}) = \frac{1}{C(\mathbf{x})} \iint_{\Psi} Z(\mathbf{x},\mathbf{y}) I(\mathbf{y}) d\mathbf{y} dt,$$

$$Z(\mathbf{x},\mathbf{y}) = g_h\left(\left(\int_{\Re^3} g_\sigma(\mathbf{t}) \left| I(\mathbf{x}-\mathbf{t}) - I(\mathbf{y}-\mathbf{t}) \right|^2 d\mathbf{t}\right)^{\frac{1}{2}}\right),$$



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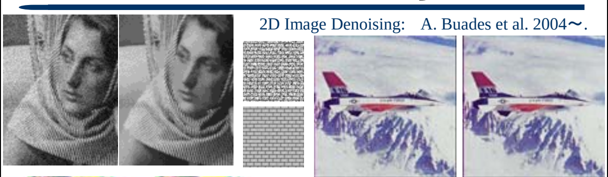
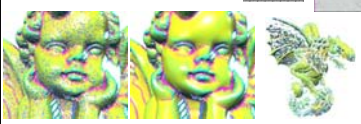
NL-Means Pattern Preserving Filters

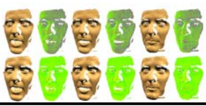
2D Image Denoising: A. Buades et al. 2004~.

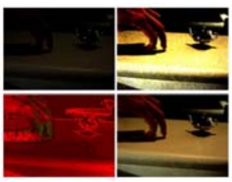




3D Mesh Smoothing: S. Yoshizawa et al. 2006.

Time-Varying Range Images Filter:
O. Schall et al. 2006.





Video Enhancement (3D Image):
E. Bennett and L.McMillan, 2005.

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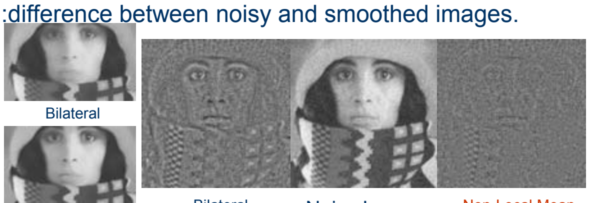
Statistical Analysis of Signal-to-Noise Ratio

- ✓ Method Noise (A. Buades, B. Coll, and J.-M. Morel): $I(\mathbf{x})^{noisy} - I^{smoothed}(\mathbf{x})$
:difference between noisy and smoothed images.



- ✓ <u>Method Noise</u>: 入力から出力を引いた画像. 特徴が出ていれば使ったFilterがノイズのみでなく, 入力画像の特徴を消している. Signal-to-Noise Ratioの空間表現
- FFTやWavelet を用いた離散Wienerフィルタとの比較は下記文

A. Buades et al. IEEE CVPR, 60-65, 2005; SIAM MMS, 4(2):490-530, 2005.

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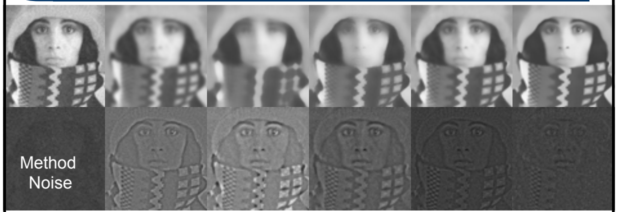
Comparisons



- ✓ Method Noise: 入力から出力を引いた画像. 特徴が出ていれば使ったFilterがノイズのみでなく, 入力画像の特徴を消している. Signal-to-Noise Ratioの空間表現
- FFTやWavelet を用いた離散Wienerフィルタとの比較は下記文

A. Buades et al. IEEE CVPR, 60-65, 2005; SIAM MMS, 4(2):490-530, 2005.

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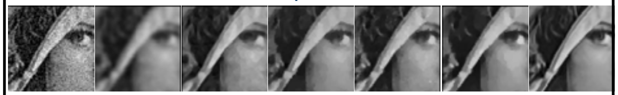
Comparisons from A Buades et al.

✓ With Gaussian, Anisotropic, Total Variations.



✓ With Wiener Filters, Wavelet thresholdings.

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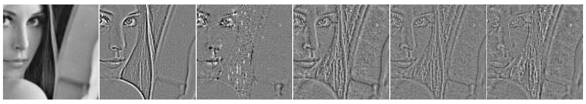
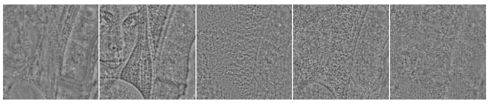
Comparisons from A Buades et al.

✓ Method Noise: $I(\mathbf{x}) - I^{new}(\mathbf{x})$





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ノイズ除去

Iterative Filteringと
領域断片化.
—発展型偏微分方程式—

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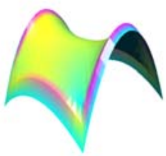
Evolutionary System of PDEs

- 発展型PDEによる変分問題の解法(非線形PDEに有用):

$$\frac{\partial I(t,\mathbf{x})}{\partial t} = F(t,\mathbf{x},I,\frac{\partial I}{\partial \mathbf{x}},\frac{\partial^2 I}{\partial \mathbf{x}^2},\ldots),$$

$$t \to \infty \Rightarrow F(\ldots) \equiv 0 \quad \text{on} \quad I(\infty,\mathbf{x}).$$

PDE: Partial Differential Equation
(偏微分方程式)



曲率モーメント最小化: Euler's Elastica

- 変分法: エネルギー最小化→ Euler-Lagrange方程式(PDE)
- 例えば, 拡散現象は…

Laplace方程式 / 拡散方程式 / 拡散過程

$$\frac{1}{2}\int |\nabla I(\mathbf{x})|^2 d\mathbf{x} \to \min \Rightarrow \Delta I(\mathbf{x}) = 0 \qquad \frac{\partial I(\mathbf{x},t)}{\partial t} = \Delta I(\mathbf{x},t), \ t \to \infty$$

Dirichletエネルギー　変分法　発展型PDEによる解法

$$\Delta I(\mathbf{x}) = \Delta I(\mathbf{x},\infty) = 0,$$

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Fair Mesh Generation via Elastica

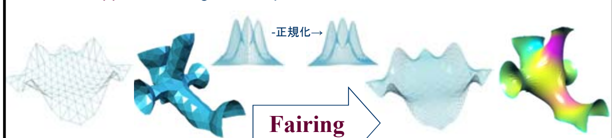
- ✓ CAD・グラフィクスアプリケーションのための意匠曲面生成
  - ✓ 美しい曲面　(Aesthetic Meshes)？
- ✓ ***Elastica Surfaces***: Criterion of Surface Fairness:

✓ The case of 2D curves was considered by L. Euler 1744.

$$\int (k_{\max}^2 + k_{\min}^2)\, dA \to \text{Min}$$

主曲率: $k_{\max}, k_{\min}$ Euler-Lagrange方程式: $\mathbf{F}$ 単位接・法線: $\mathbf{T},\mathbf{N}$

- 難しさ: 高階の非線形偏微分方程式 (安定性)
  - Generalized Willmore Flow: $\mathbf{S}_t = \mathbf{F}\,\mathbf{N} + G\mathbf{T}.$
  - Our Approach: Tangent Component G によるメッシュの正規化.



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Iterative特徴保存フィルタ

- Neighbornood Filteringも発展型PDEによる記述が可能:

$$I^{new}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y})I(\mathbf{y})dy}{\int Z(\mathbf{x},\mathbf{y})dy}$$

繰り返しによる収束

$$F(\ldots) = \frac{\int Z(\mathbf{x},\mathbf{y})I(\mathbf{y})dy}{\int Z(\mathbf{x},\mathbf{y})dy} - I^{new}(\mathbf{x}) = 0$$

$$\frac{\partial I(t,\mathbf{x})}{\partial t} = F(t,\mathbf{x},\ldots) = \frac{\int Z(t,\mathbf{x},\mathbf{y})I(t,\mathbf{y})dy}{\int Z(t,\mathbf{x},\mathbf{y})dy} - I(t,\mathbf{x}),$$

発展型PDE

$$I^{n+1}(\mathbf{x}) = I^{n}(\mathbf{x}) + \varepsilon\left(\frac{\int Z(n,\mathbf{x},\mathbf{y})I^{n}(\mathbf{y})dy}{\int Z(n,\mathbf{x},\mathbf{y})dy} - I^{n}(\mathbf{x})\right),$$

一次陽差分解法 ⇒ 局所的な繰り返しフィルタ

$$I^{n+1}(\mathbf{x}) = I^{n}(\mathbf{x}) + \varepsilon\left(\frac{\int Z(n,\mathbf{x},\mathbf{y})I^{n+1}(\mathbf{y})dy}{\int Z(n,\mathbf{x},\mathbf{y})dy} - I^{n+1}(\mathbf{x})\right),$$

一次準陰差分解法 ⇒ $Ax=b$ 疎な連立一次方程式

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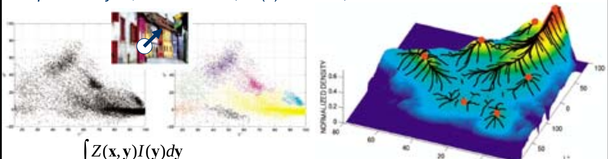
Mean Shift and Compact Support Filtering

- From D. Comaniciu and P. Meer, *Mean Shift: A Robust Approach Toward Feature Space Analysis,* In IEEE PAMI, 24(5): 603-619, 2002.



$$I^{new}(\mathbf{x}) = \frac{\int Z(\mathbf{x},\mathbf{y})I(\mathbf{y})dy}{\int Z(\mathbf{x},\mathbf{y})dy}$$

D. Comaniciu and P. Meer ©IEEE.

$$\mathbf{p}^{0} = \mathbf{q}, \quad \mathbf{q} = \{\mathbf{x}, I(\mathbf{x})\}$$

$$\mathbf{p}^{n+1} = \frac{\int Z(\mathbf{q},\mathbf{y})\mathbf{q}dy}{\int Z(\mathbf{q},\mathbf{y})dy}$$

- See also R. Boomgaard and J. Weijer, *On the Equivalence of Local-Mode Finding, Robust Estimation and Mean-Shift Analysis as used in Early Vision Tasks,* In ICPR, 3: 30927-30930, 2002.

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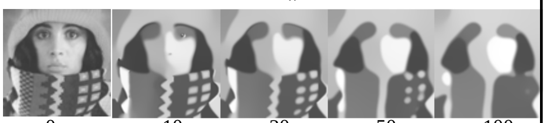
Joint Filtering

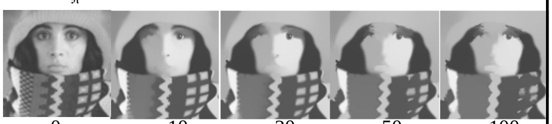
発展型NL-Means Filter: $D(t,\mathbf{x},\mathbf{y}) = \int_{\Re^{l}} g_{\sigma}(\mathbf{t})\left|I(t,\mathbf{x}-\mathbf{t}) - I(t,\mathbf{y}-\mathbf{t})\right|^{2} d\mathbf{t}$



n: 0 10 20 50 100

発展型Joint NL-Means Filter:

$$D(t,\mathbf{x},\mathbf{y}) = \int_{\Re^{l}} g_{\sigma}(\mathbf{t})\left|I(t,\mathbf{x}-\mathbf{t}) - q(t,\mathbf{y}-\mathbf{t})\right|^{2} d\mathbf{t}, \quad e.g.\ q(t,\mathbf{x}) = I(0,\mathbf{x}).$$



n: 0 10 20 50 100

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研究・応用:
生物・医用画像のノイズ除去

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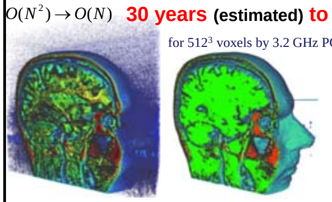
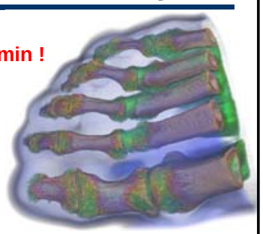
Iterative Bilateral & NL-Means for Biomedical Images

新しいBilateral Filter高速計算法の提案

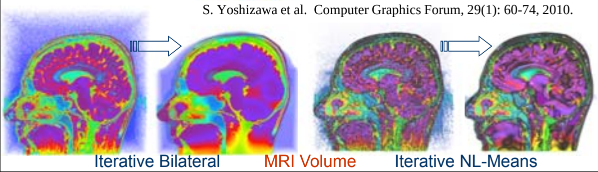
$O(N^2) \rightarrow O(N)$ 30 years (estimated) to 3 min !

for 512³ voxels by 3.2 GHz PC.





S. Yoshizawa et al. Computer Graphics Forum, 29(1): 60-74, 2010.



Iterative Bilateral MRI Volume Iterative NL-Means

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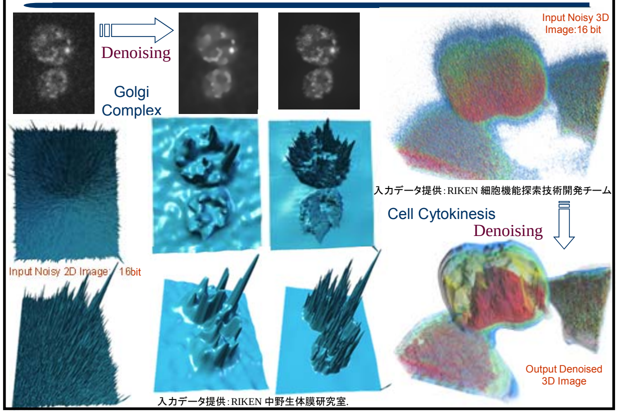
Iterative NL-Means for Intracellular Images



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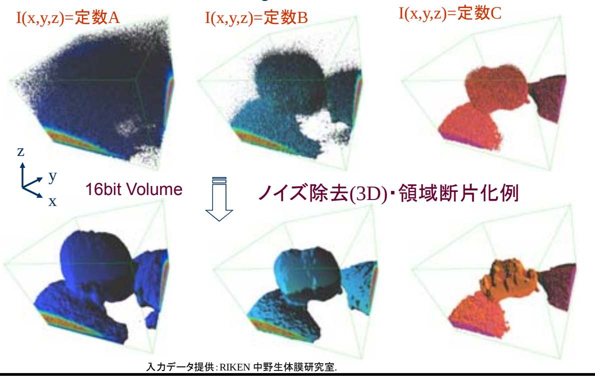
Iterative NL-Means for Intracellular Images

- ✓ Iterative NL-Means Filteringの結果.



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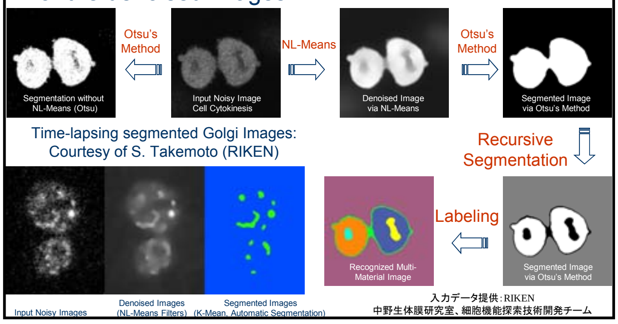
Iterative NL-Means is useful for segmentation !

- ✓ Since NL-Means filter emphasizes pattern boundaries, conventional image segmentation methods work well for the denoised images.



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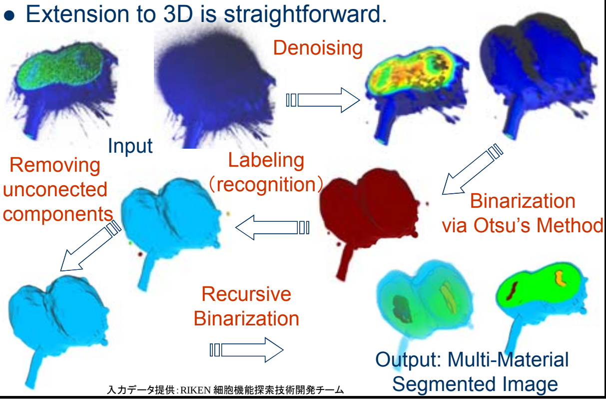
Automatic Segmentation to 3D Image

- Extension to 3D is straightforward.



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有用な参考文献＆URL

- ✓ ***Bilateral Filter入門:***
  S. Paris, et al. A gentle introduction to bilateral filtering and its applications. In ACM SIGGRAPH '07 courses, IEEE CVPR' 08 Tutorials, and ACM SIGGRAPH Asia' 08 classes.
- ✓ **MITの高速Bilateral Filterのページ:**
  http://people.csail.mit.edu/sparis/bf/
  上記入門のPDF、参考文献多数、高速プログラム(ソースコードあり、FFT-basedの2D、Color 5D、及びTone Mapping)

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研究・応用:
メラノソーム
直径推定

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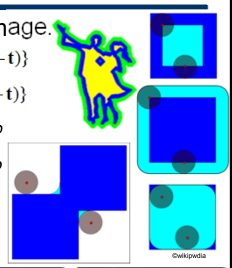
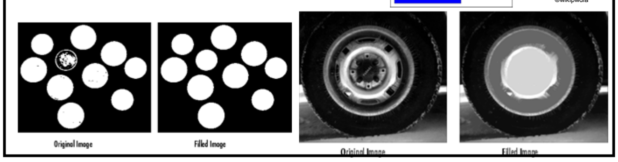
Morphologicalフィルタ

- ✓ Structuring Element (b): Binary image.
- ✓ Erosion(収縮): $[f-b](\mathbf{x}) = \min_{\mathbf{t}\in b}\{f(\mathbf{x}-\mathbf{t})\}$
- ✓ Dilation(膨張): $[f+b](\mathbf{x}) = \max_{\mathbf{t}\in b}\{f(\mathbf{x}-\mathbf{t})\}$
- ✓ Opening(穴あけ): $(f \circ b) = (f-b)+b$
- ✓ Closing(穴埋め): $(f \bullet b) = (f+b)-b$
- ✓ Top Hat: $T_{hat}(f) = f-(f \circ b)$
- ✓ Bottom Hat: $B_{hat}(f) = (f \bullet b)-f$





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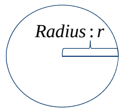
Morphologicalフィルタ2

- Structuring Element (b): Binary image.
  - ベシクル、～ソーム等の輸送担体:Disk with radius r.
- Erosion(収縮): $[f - b](\mathbf{x}) = \min_{\mathbf{t}\in b}\{f(\mathbf{x}-\mathbf{t})\}$
- Dilation(膨張): $[f + b](\mathbf{x}) = \max_{\mathbf{t}\in b}\{f(\mathbf{x}-\mathbf{t})\}$
- Opening(穴あけ): $(f \circ b) = (f - b) + b$
- Closing(穴埋め): $(f \bullet b) = (f + b) - b$
- Top Hat: $T_{hat}(f) = f - (f \circ b)$
- Bottom Hat: $B_{hat}(f) = (f \bullet b) - f$
- Granulometry: $g(r) = \partial \sum_{pixels} f \circ b(r) / \partial r$



Disk Binary Image

Granulometryは画像内のdisk-like形状物体の大きさとその数の分布を個々の領域抽出なしでラフに見積もれる！

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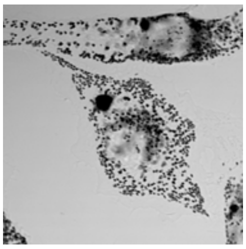
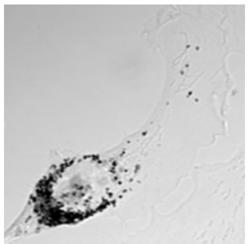
メラノソームの動態



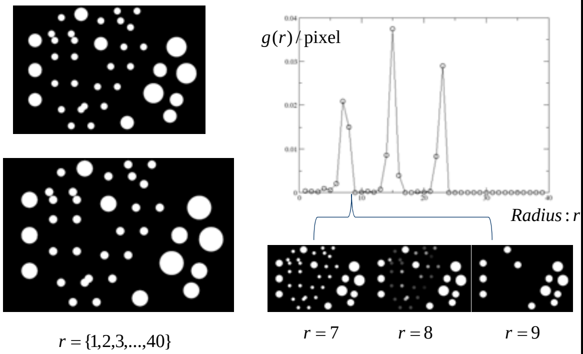


データ提供:東北大学：福田先生

Shin Yoshizawa: shin@riken.jp

Granulometry

- Granulometry: $g(r) = \partial \sum_{pixels} f \circ b(r) / \partial r$



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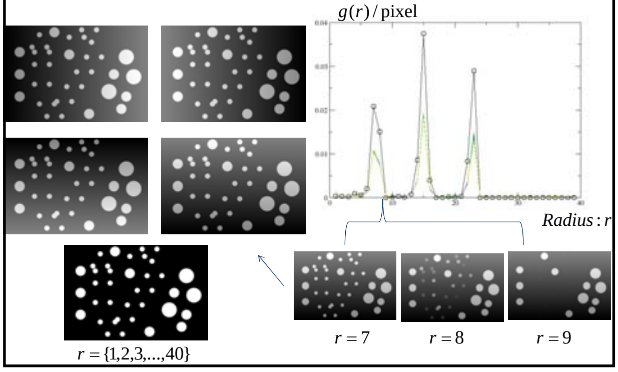
Granulometry: Shading Robust

- Granulometry: $g(r) = \partial \sum_{pixels} f \circ b(r) / \partial r$



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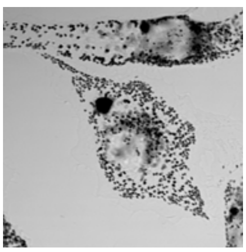
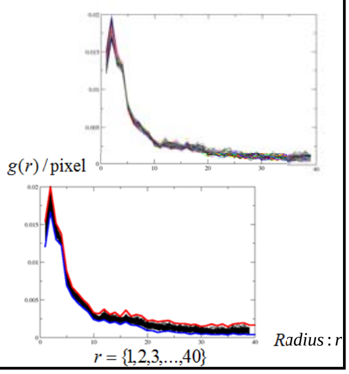
Granulometry: Case Study

- 正常時メラノソーム:





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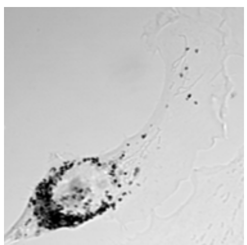
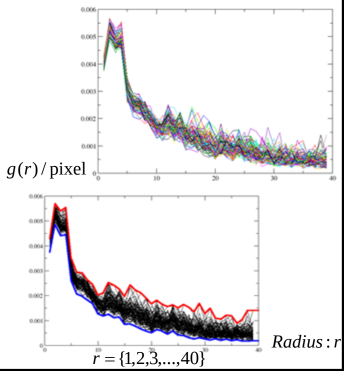
Granulometry: Case Study

- Griscelli症候群メラノソーム:





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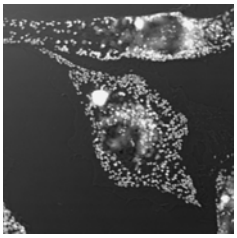
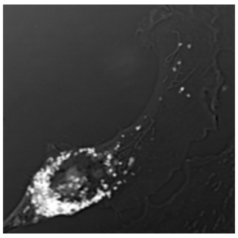
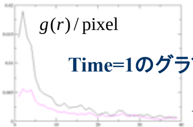
Granulometry: Case Study

✓ メラノソーム:







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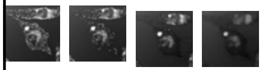
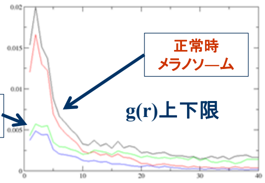
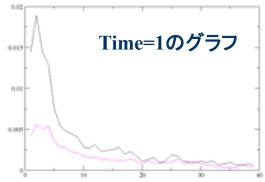
Granulometry: Case Study

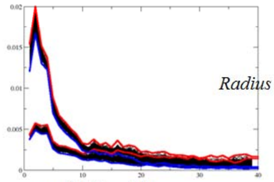
✓ メラノソーム:



r=2,4,8,16のVideo







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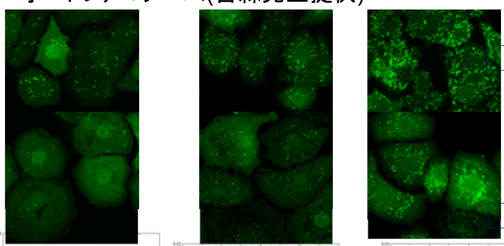
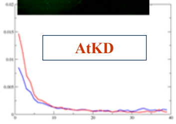
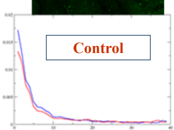
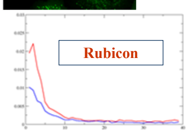
Granulometry: Case Study

✓ オートファゴソーム(吉森先生提供)









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Current Limitation & Future Work

✓ 空間移動に対する変化がない。

✓ ノイズ比が高い場合にRobustではない。

時間変化画像へのGranulometry又は周波数分解後のGranulometry

$g(r)=\partial\sum_{\text{pixels}} f\circ b(r)/\partial r$ ⇒ $g(r)=\partial\sum_{\text{pixels}} f_{low/high}\circ b(r)/\partial r$

$$g(r)=\partial\sum_{\text{pixels}} h(f,\frac{\partial f}{\partial t})\circ b(r)/\partial r$$

✓様々な統計量での評価及び相関のCross-Validationによる評価

✓3D化は簡単！

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まとめ

- ✓ 生物・医用画像、平均化、メディアン、加算ノイズ、白色性、ガウス性、ウィナーフィルタ、逆畳み込み.
- ✓ 平滑化フィルタ:
  - Bilateralフィルタ(エッジ保存).
  - 高速Bilateralフィルタ:Separable、FFT、ヒストグラム、FGT.
  - 周波数分解によるTone Mapping.
  - Non-Local Meansフィルタ(パターン保存).
  - フィルタの再帰的適用=発展型偏微分方程式.
  - 断片化と領域抽出.
  - Morphologicalオペレータ、Granulometry.