# Topological current at an interface between superfluid $^3$He A-phases

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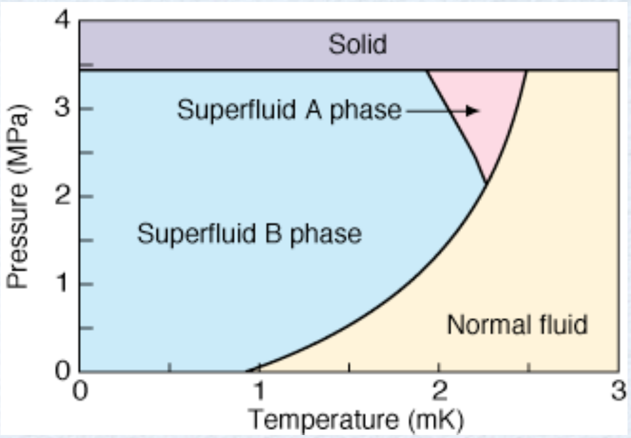
# Edge state and domain wall




**A-phase** high-temperature and high-pressure

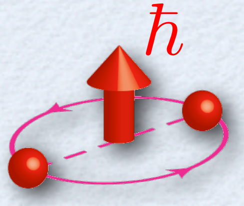
Anderson-Brinkman-Morel (ABM) state

spin state $S_z = 0$ $\boldsymbol{d}$

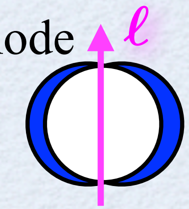
orbital state $L_z = +1$ $\boldsymbol{\ell}$

$d_z(k_x + i k_y)$

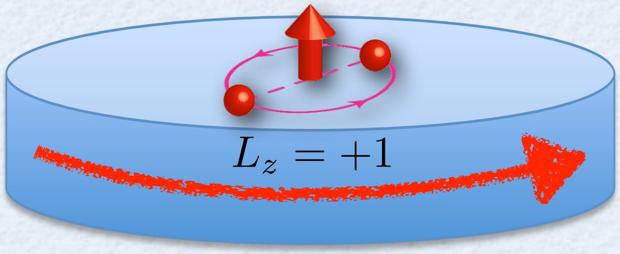
broken time-reversal symmetry




point node



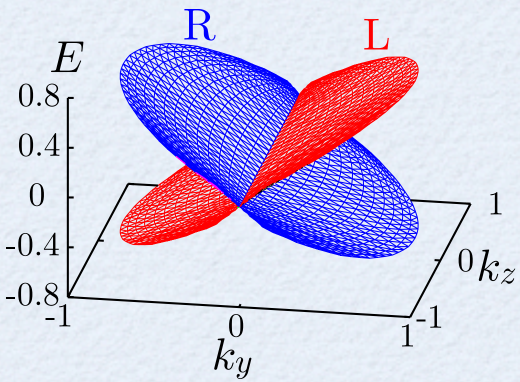

## Edge state




$J = \frac{n\hbar}{4}$

$L_z = \frac{N\hbar}{2}$

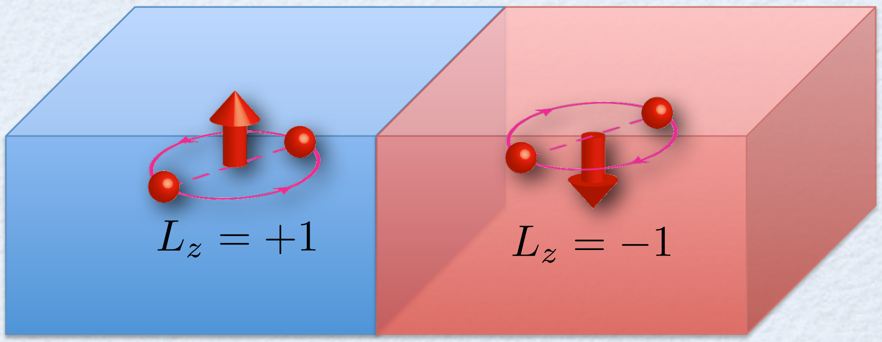
macroscopic intrinsic angular momentum




$n$ : density of $^3$He atoms

$N$ : number of $^3$He atoms

## Domain wall




topological mass current

# Quasi-classical Eilenberger theory

$\Delta/E_F \ll 1$ $\quad$ $$\int d\xi_k \widehat{\sigma}_z \widehat{G}(\boldsymbol{k},\boldsymbol{r},\omega_n) \equiv \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix}$$

**Eilenberger equation**

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) = \left[ \begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F,\boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \right]$$

$\widehat{g}$ $\qquad$ $\hat{\Delta}$

**Gap equation**

$$\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) = N_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \left\langle V(\boldsymbol{k}_F,\boldsymbol{k}'_F) \hat{f}(\boldsymbol{k}'_F,\boldsymbol{r},\omega_n) \right\rangle_{\boldsymbol{k}'_F}$$

pair potential : $V(\boldsymbol{k}_F,\boldsymbol{k}'_F) = 3g_1 \boldsymbol{k}_F \cdot \boldsymbol{k}'_F$

mass current: $\boldsymbol{j}(\boldsymbol{r},T) = mN_0\pi k_B T \sum_{\omega_n} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)]\rangle_{\boldsymbol{k}_F}$

dispersion: $N(\boldsymbol{k}_F,\boldsymbol{r},E) = N_0 \mathrm{Re}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n \to E+i\eta}]$

# Riccati method

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \left[ \begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F, \boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \right]$$

$$\widehat{g} \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix}$$

$$\begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} = \begin{pmatrix} (\hat{1} + \hat{a}\hat{b})^{-1} & 0 \\ 0 & (\hat{1} + \hat{b}\hat{a})^{-1} \end{pmatrix} \begin{pmatrix} \hat{1} - \hat{a}\hat{b} & 2i\hat{a} \\ -2i\hat{b} & -(\hat{1} - \hat{b}\hat{a}) \end{pmatrix}$$

**Riccati equations**

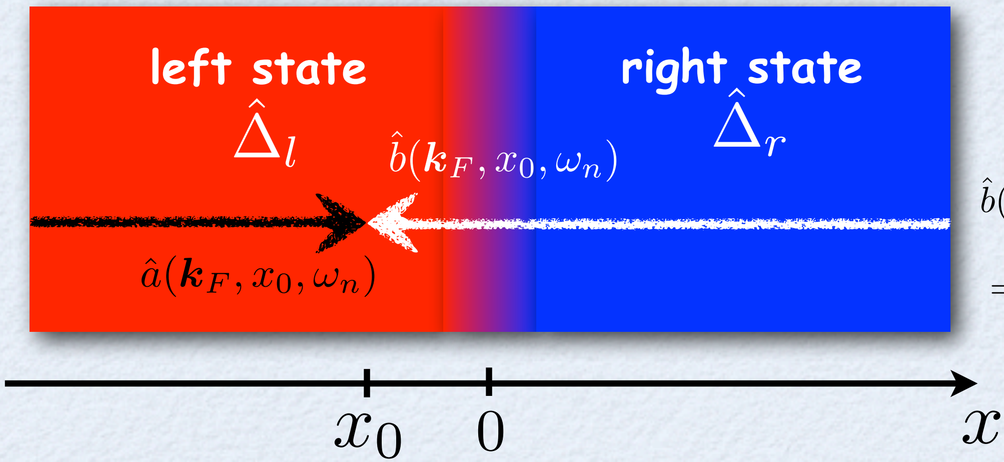
toward $\boldsymbol{v}_F$

$$\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta} - \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \hat{\Delta}^\dagger \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

toward $-\boldsymbol{v}_F$

$$-\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta}^\dagger - \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \hat{\Delta} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

$$\hat{a}(\boldsymbol{k}_F, -\infty, \omega_n) = \frac{\hat{\Delta}_l}{\omega_n + \sqrt{\omega_n^2 + |\Delta_l|^2}}$$




$$\hat{b}(\boldsymbol{k}_F, +\infty, \omega_n) = \frac{\hat{\Delta}_r^\dagger}{\omega_n + \sqrt{\omega_n^2 + |\Delta_r|^2}}$$

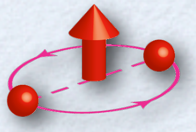
$$|\Delta|^2 = \frac{1}{2} \mathrm{Tr} \left( \hat{\Delta} \hat{\Delta}^\dagger \right)$$

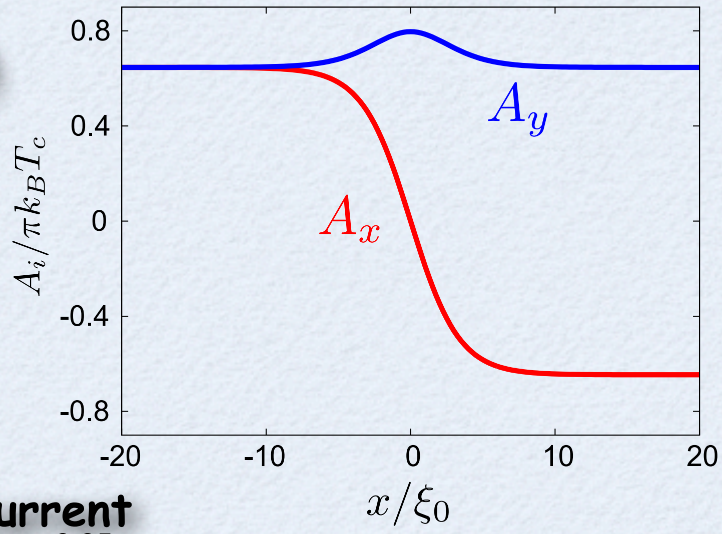
# Result (edge state)

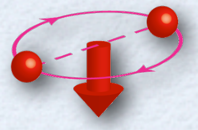
$T = 0.2T_c$

$\Delta = d_z(A_x k_x + i A_y k_y)$

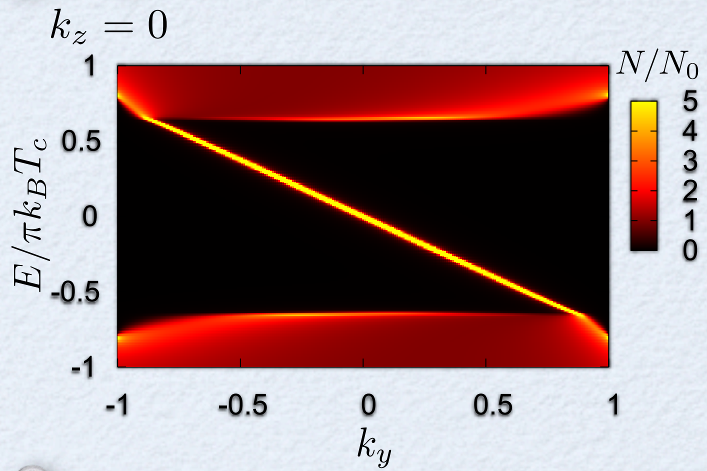
$\Delta_l = d_z(k_x + ik_y)$ same with edge state $\Delta_r = d_z(-k_x + ik_y)$



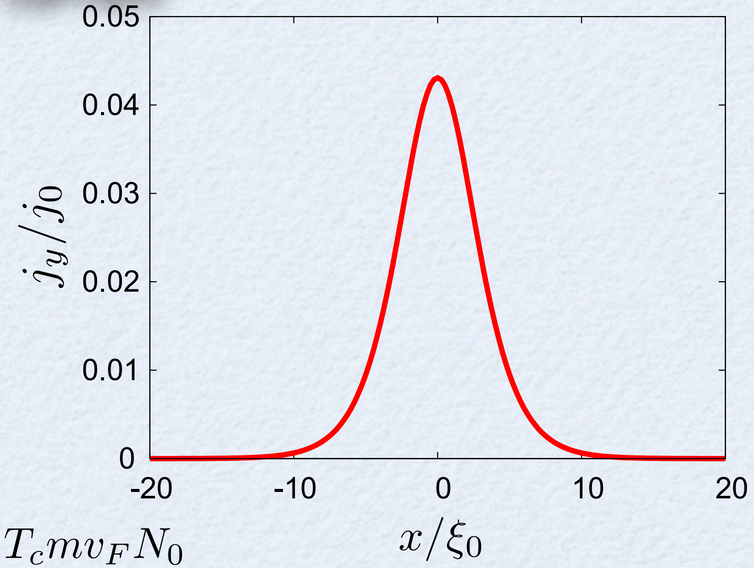




## dispersion at domain wall



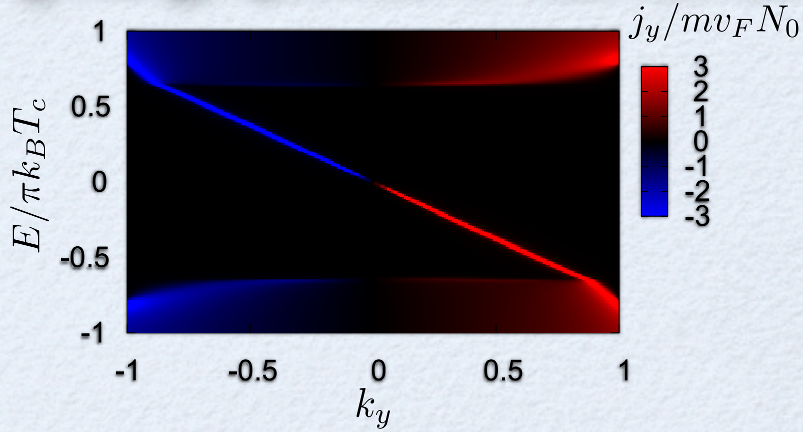

## mass current




$J_y \approx \frac{n\hbar}{4}$

$j_0 = \pi k_B T_c m v_F N_0$

## energy spectrum of mass current

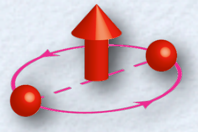


# Result (lowest energy domain)

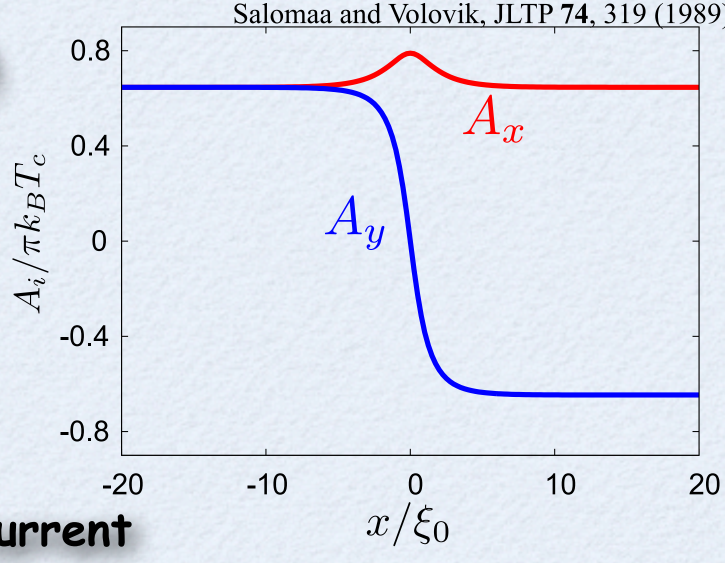
$$\Delta = d_z(A_x k_x + iA_y k_y)$$

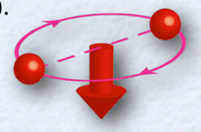
$\Delta_l = d_z(k_x + ik_y)$ lowest energy $\Delta_r = d_z(k_x - ik_y)$

Salomaa and Volovik, JLTP **74**, 319 (1989).



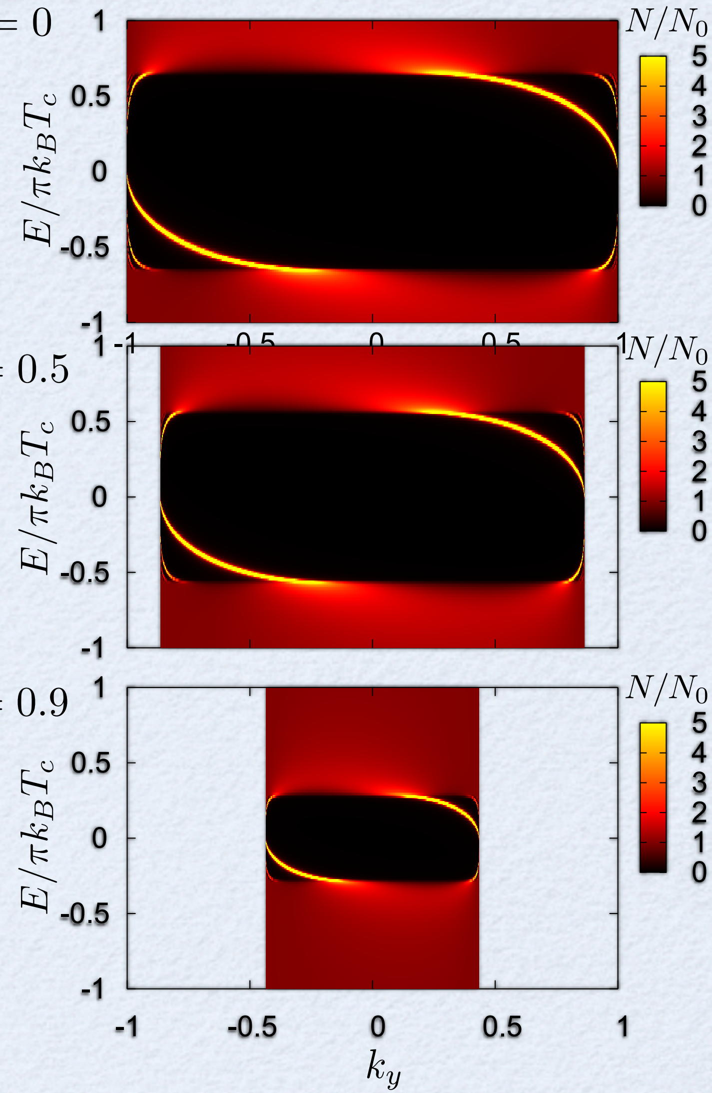
$L_z = +1$



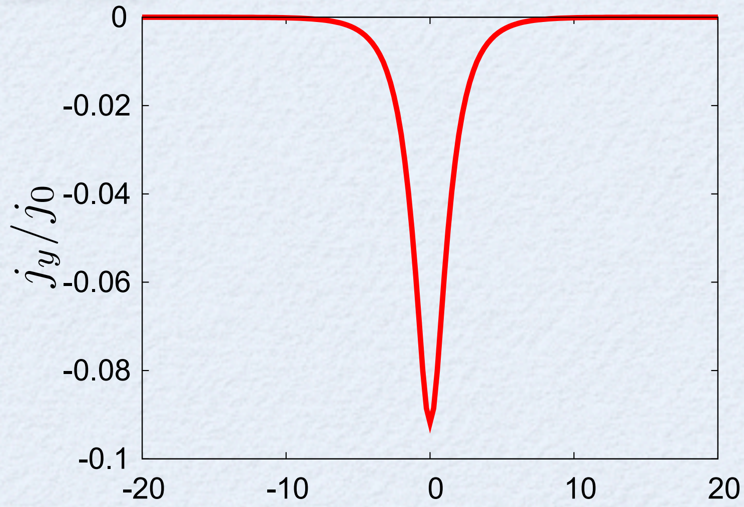



$L_z = -1$

**dispersion at domain wall**



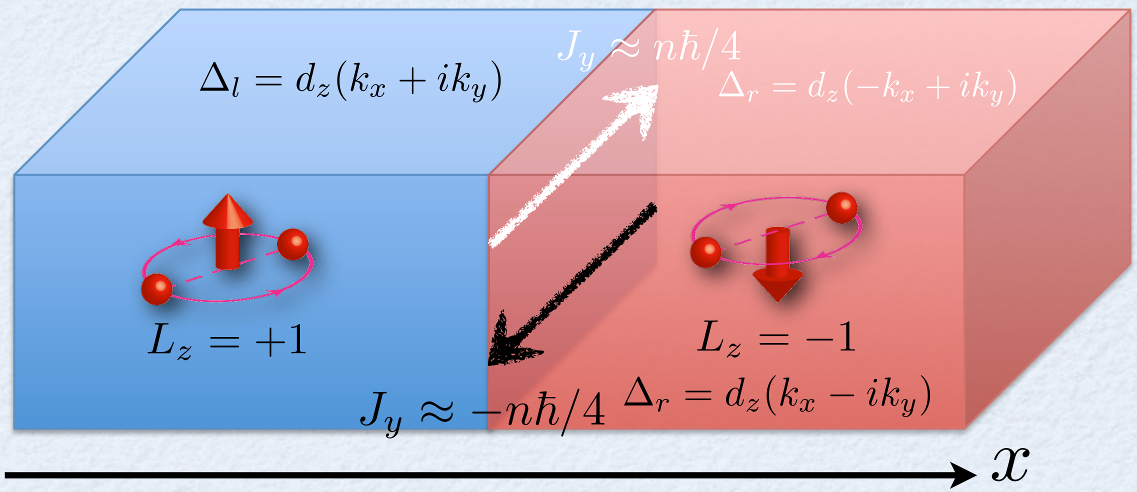

**mass current**



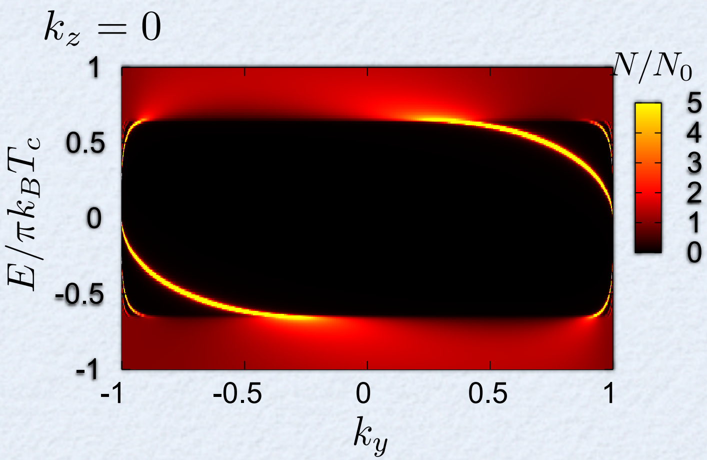

$$J_y \approx -\frac{n\hbar}{4}$$

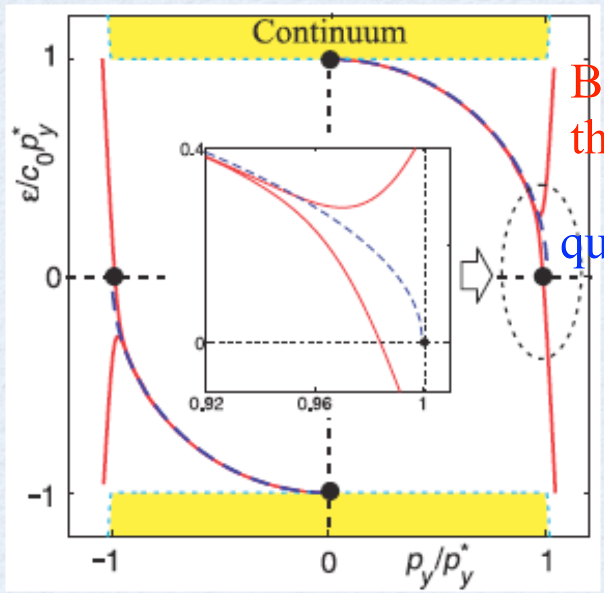
$j_0 = \pi k_B T_c m v_F N_0$

# Summary and remark




**Topological mass current has no relation to intrinsic angular momentum.**




Bogoliubov-de Gennes theory

quasiclassical theory

by normal reflection

M. A. Silaev and G. E. Volovik, PRB **86**, 214511 (2012).