Topological mass current on a domain wall in superfluid ^{3}He A-phase

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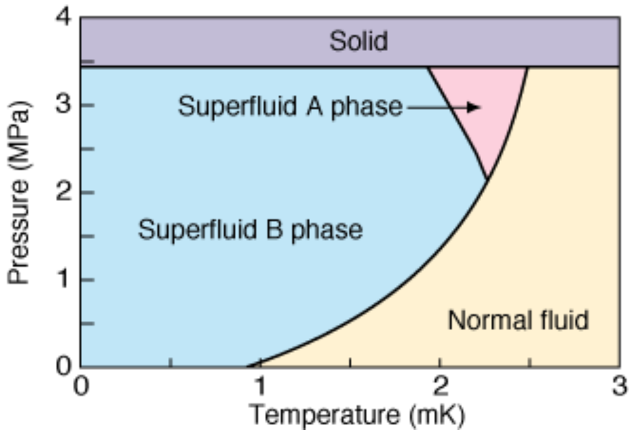
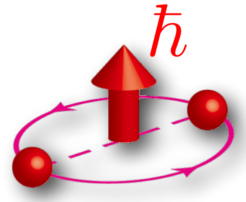
Domain wall in A-phase



A-phase high-temperature and high-pressure

Anderson-Brinkman-Morel (ABM) state

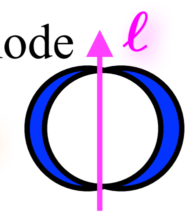
spin state $S_z = 0$ $\boldsymbol{d}$



equal spin pairing: $|\uparrow\uparrow\rangle,\ |\downarrow\downarrow\rangle$

orbital state $L_z = \pm 1$ $\boldsymbol{\ell}$

point node



broken time-reversal symmetry

$$\boldsymbol{d} = \Delta_0 \boldsymbol{z}(\pm k_x + i k_y)$$

topological number

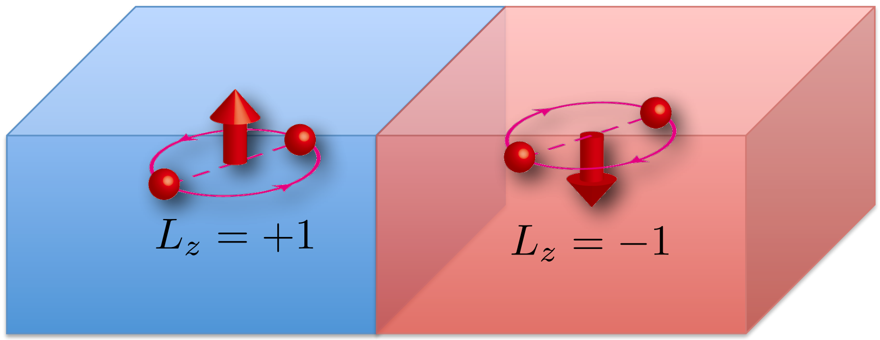
$$\nu = \underline{2}\ \mathrm{sgn}(\Delta_x)\ \mathrm{sgn}(\Delta_y)$$

spin states

$$= \pm 2$$

Domain wall

$$\nu_{\mathrm{L}} = +2 \qquad \nu_{\mathrm{R}} = -2$$



topological phase transition

zero energy bound state

mass current

$$\Delta_0 \boldsymbol{z}(k_x + i k_y)$$

$$\Delta_0 \boldsymbol{z}(-k_x + i k_y)$$

and

$$-\Delta_0 \boldsymbol{z}(-k_x + i k_y)$$

Quasi-classical Eilenberger theory

$\Delta/E_F \ll 1$

$$\int d\xi_k \widehat{\sigma}_z \widehat{G}(\boldsymbol{k},\boldsymbol{r},\omega_n) \equiv \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix}$$

Eilenberger equation

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) \\ \hat{\Delta}^{\dagger}(\boldsymbol{k}_F,\boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n) \right]$$

$\widehat{g}$ ↓ ↑ $\hat{\Delta}$

Gap equation

$$\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) = N_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \left\langle V(\boldsymbol{k}_F,\boldsymbol{k}'_F) \hat{f}(\boldsymbol{k}'_F,\boldsymbol{r},\omega_n) \right\rangle_{\boldsymbol{k}'_F}$$

pair potential : $V(\boldsymbol{k}_F,\boldsymbol{k}'_F) = 3g_1 \boldsymbol{k}_F \cdot \boldsymbol{k}'_F$

mass current: $\boldsymbol{j}(\boldsymbol{r},T) = mN_0\pi k_B T \sum_{\omega_n} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)] \rangle_{\boldsymbol{k}_F}$

dispersion: $N(\boldsymbol{k}_F,\boldsymbol{r},E) = N_0 \mathrm{Re}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n \to E+i\eta}]$

Riccati method

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F, \boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \right]$$

$$\widehat{g} \equiv -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix}$$

$$\begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} = \begin{pmatrix} (\hat{1} + \hat{a}\hat{b})^{-1} & 0 \\ 0 & (\hat{1} + \hat{b}\hat{a})^{-1} \end{pmatrix} \begin{pmatrix} \hat{1} - \hat{a}\hat{b} & 2i\hat{a} \\ -2i\hat{b} & -(\hat{1} - \hat{b}\hat{a}) \end{pmatrix}$$

Riccati equations

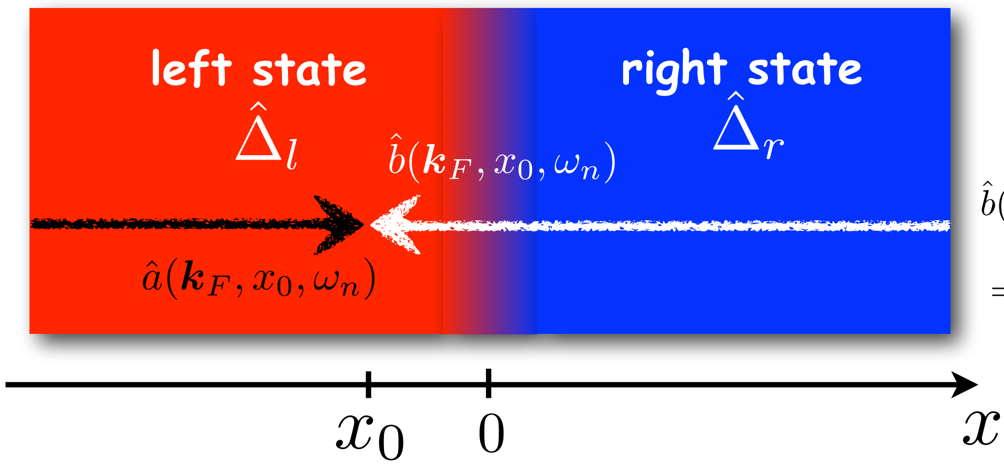
toward $\boldsymbol{v}_F$

$$\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta} - \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \hat{\Delta}^\dagger \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

toward $-\boldsymbol{v}_F$

$$-\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta}^\dagger - \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \hat{\Delta} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

$$\hat{a}(\boldsymbol{k}_F, -\infty, \omega_n) = \frac{\hat{\Delta}_l}{\omega_n + \sqrt{\omega_n^2 + |\Delta_l|^2}}$$



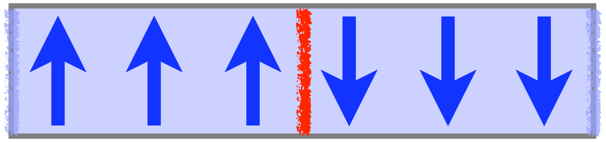
$$\hat{b}(\boldsymbol{k}_F, +\infty, \omega_n) = \frac{\hat{\Delta}_r^\dagger}{\omega_n + \sqrt{\omega_n^2 + |\Delta_r|^2}}$$

$$|\Delta|^2 = \frac{1}{2} \mathrm{Tr} \left(\hat{\Delta} \hat{\Delta}^\dagger \right)$$

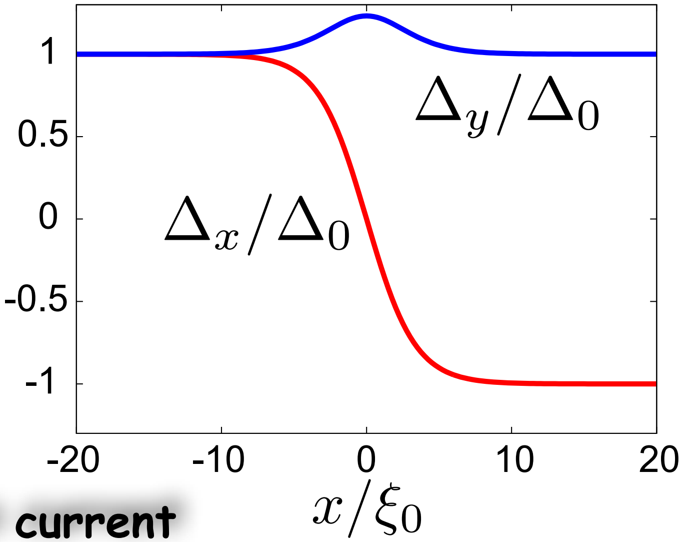
Result

$\Delta_0 \boldsymbol{z}(k_x + ik_y)$ $\quad$ $\Delta_0 \boldsymbol{z}(-k_x + ik_y)$

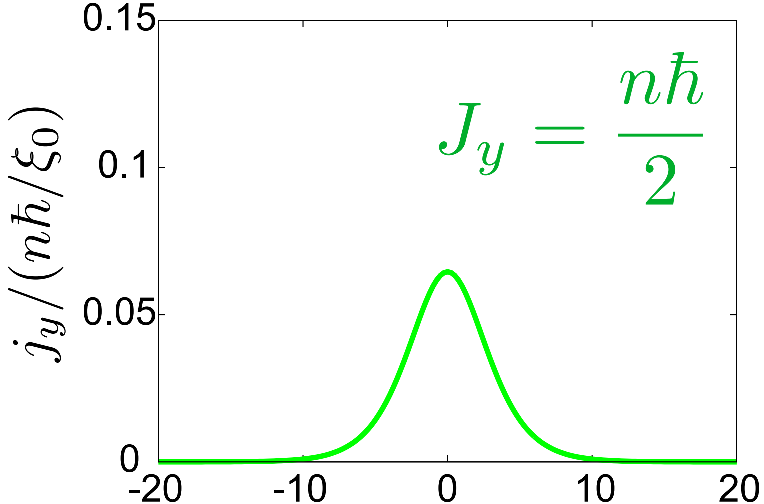
$\nu_{\rm L} = +2$ $\quad$ $\nu_{\rm R} = -2$



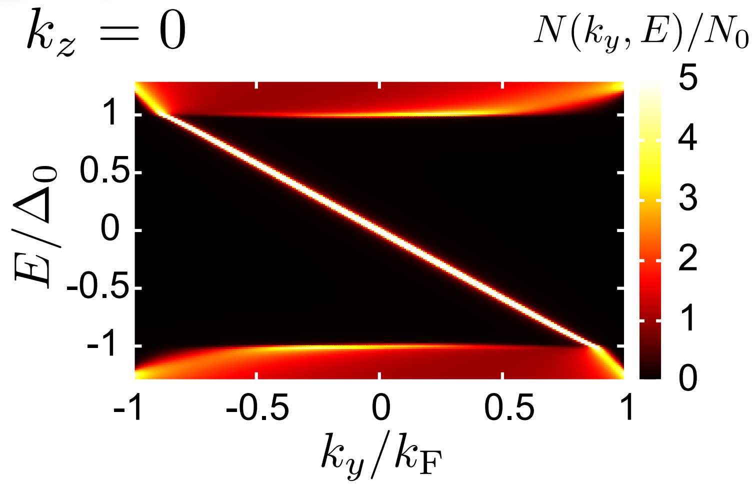
order parameter



mass current



dispersion at domain wall



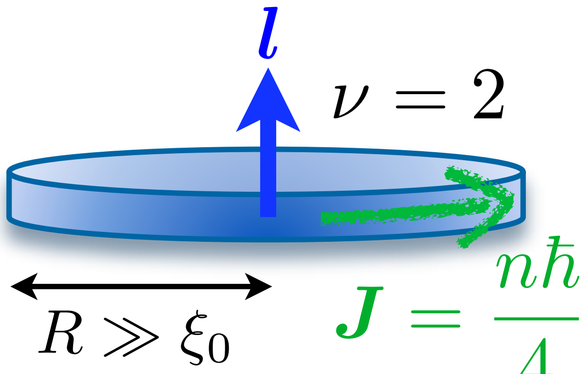
$$\Delta\nu = 2(\nu_{\rm R} - \nu_{\rm L}) = \sum_{E=0} {\rm sgn}\left[\frac{\partial E}{\partial k_y}\right]$$

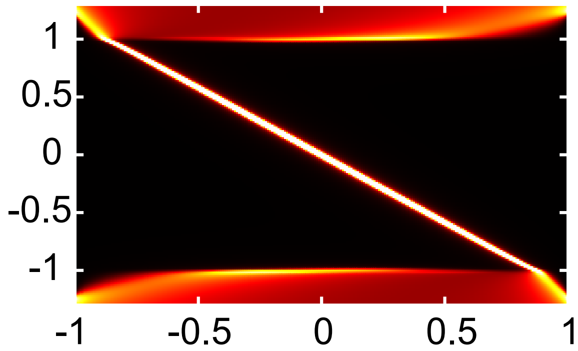
$$= -4$$

spin degeneracy

contribution from each domain

Cf. edge bound state





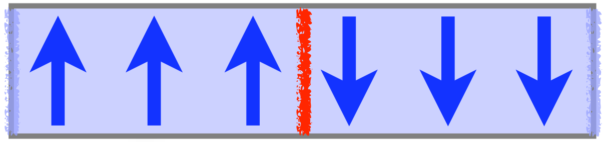
$\Delta\nu = -\nu = -2$

spin degeneracy

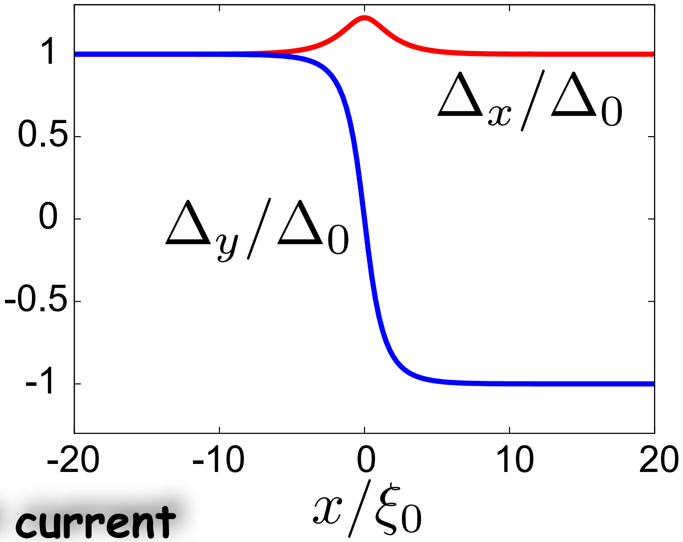
Result

$\Delta_0 \boldsymbol{z}(k_x + ik_y)$ $-\Delta_0 \boldsymbol{z}(-k_x + ik_y)$

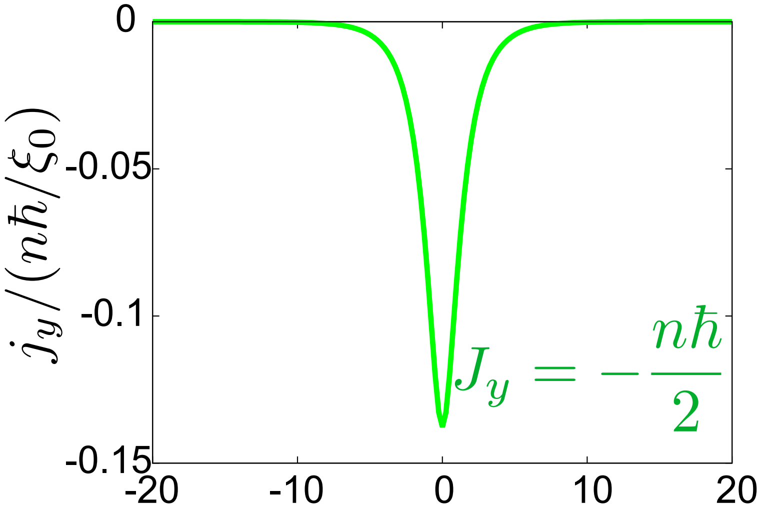
$\nu_{\mathrm{L}} = +2$ $\nu_{\mathrm{R}} = -2$



order parameter

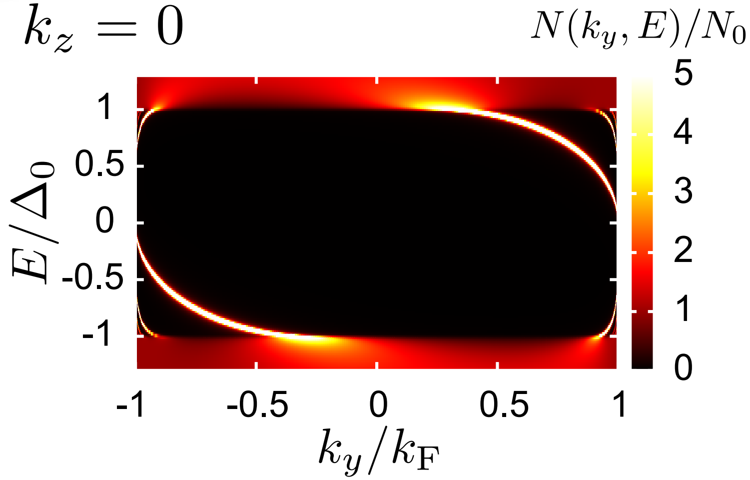


mass current



dispersion at domain wall

$k_z = 0$

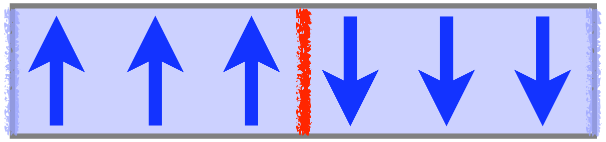


$\Delta\nu = \nu_{\mathrm{R}} - \nu_{\mathrm{L}} = -4$

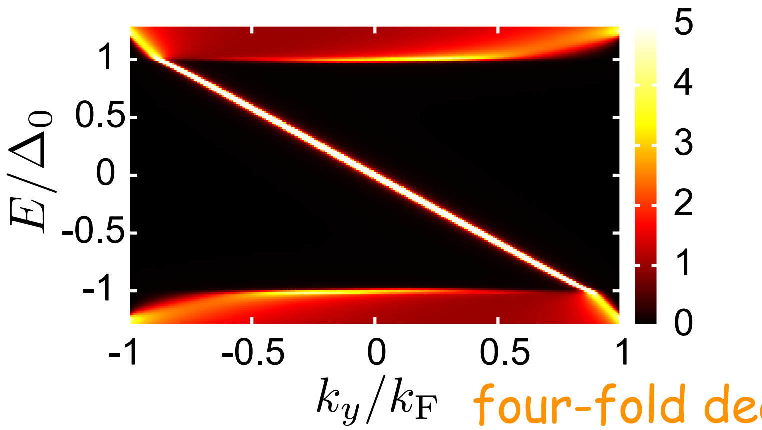
spin degeneracy

Discussion

$\Delta_0 \boldsymbol{z}(k_x + ik_y)$ $\Delta_0 \boldsymbol{z}(-k_x + ik_y)$



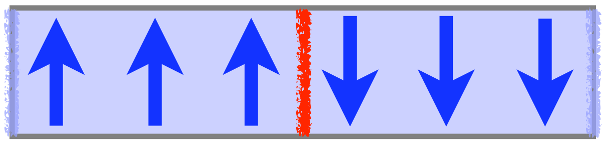
$J_y = \frac{n\hbar}{2}$



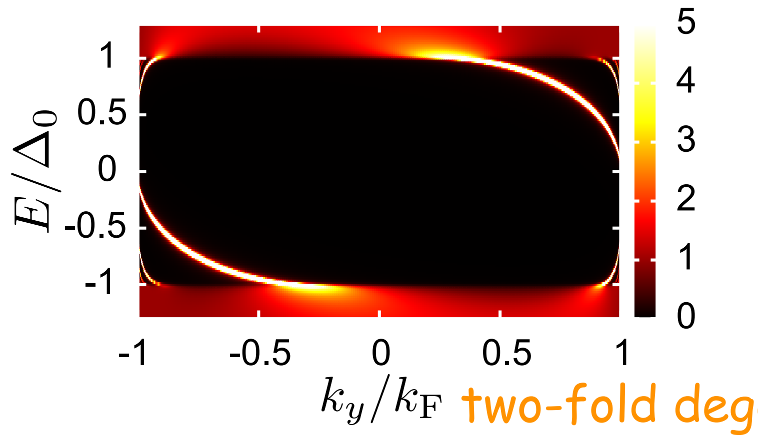
four-fold degeneracy

π-phase shift: $(k_x, k_y) = (\pm k_{\rm F}, 0)$

$\Delta_0 \boldsymbol{z}(k_x + ik_y)$ $-\Delta_0 \boldsymbol{z}(-k_x + ik_y)$



$J_y = -\frac{n\hbar}{2}$



two-fold degeneracy

π-phase shift: $(k_x, k_y) = (0, \pm k_{\rm F})$

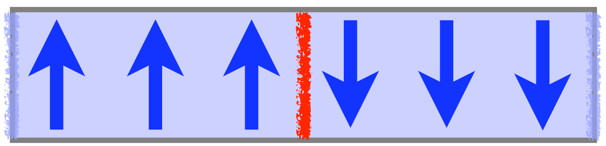
$$\Delta J_y = \frac{n\hbar}{4} \sum_{E=0} \left(\frac{k_y}{k_F} \right)^2 {\rm sgn}\left[\frac{\partial E}{\partial k_y} \right]$$

$$= -n\hbar$$

G. E. Volovik, Pis'ma ZhETF **66**, 492 (1997).

general domain wall

$\Delta_0 \boldsymbol{z}(k_x + ik_y)$ $e^{i\phi}\Delta_0 \boldsymbol{z}(-k_x + ik_y)$



π-phase shift: $(k_x, k_y) = \pm k_{\rm F}(\cos[\phi/2], \sin[\phi/2])$

for $\phi = \pm\pi/2$, $(k_x, k_y) = \pm k_{\rm F}(1/\sqrt{2}, \pm 1/\sqrt{2})$

$\Delta J_y = -n\hbar/2$ → $J_y = 0$

Summary and prospects

Summary

I have studied mass current on domain walls
between A-phases with opposite direction of angular momentum.

Mass current flows toward +y-direction with changing the sign of k_x-component,
but it flows toward -y-direction with changing the sign of k_y-component.

The difference is originated from momentum of π-phase shift
through spectral flow.

Future prospects

$$\Delta_0 z(k_x + ik_y) \; / \; \pm i\Delta_0 z(-k_x + ik_y)$$

Is the domain wall without mass current realized and most stable?