16P-A008

On Intrinsic Angular Momentum due to Edge Mass Current for Superfluid ^{3}He A-Phase

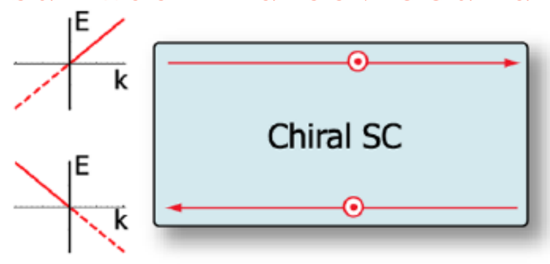
Department of Physics, Okayama University

Y. Tsutsumi, T. Mizushima, M. Ichioka, and K. Machida

Introduction

Edge mass current

surface Andreev bound state



X.-L. Qi, *et al.*, PRL **102**, 187001 (2009).

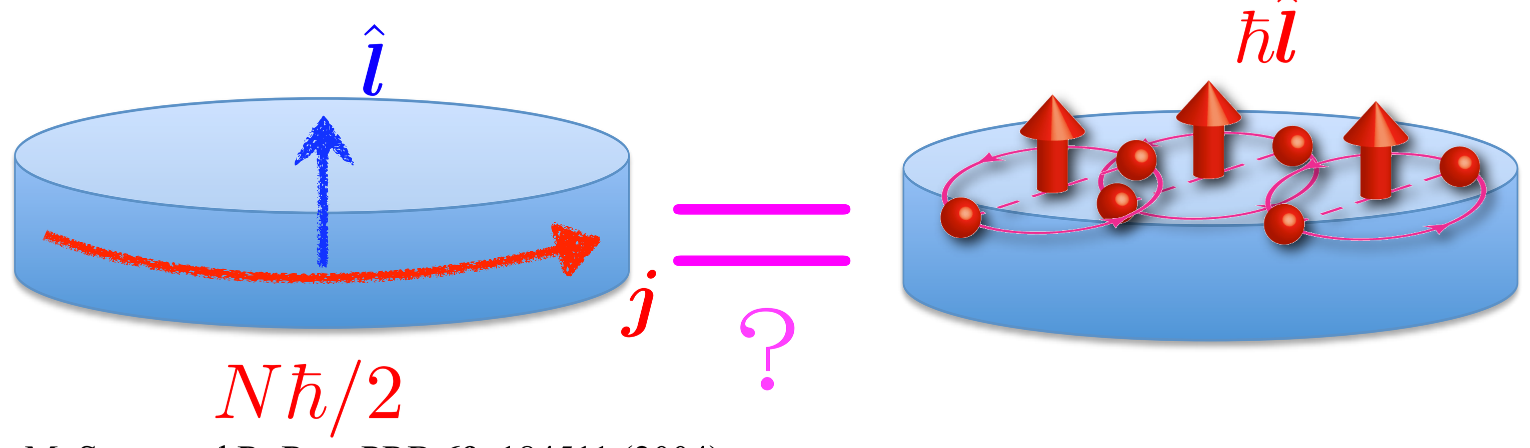
Intrinsic angular momentum

$L \sim N\hbar \left(\frac{\Delta}{E_F}\right)^\gamma \quad \gamma = 0, 1, 2\ ?$

N : total number of ^{3}He atoms

If $\gamma = 0$, $L = N\hbar/2$

M. Ishikawa, Prog. Theor. Phys. **57**, 1836 (1977).



M. Stone and R. Roy, PRB **69**, 184511 (2004).

- Contribution to edge mass current from bound state and continuum
- Temperature dependence of angular momentum compared with superfluid density

Quasi-Classical Theory

Eilenberger equation

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla}\hat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F, \boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \hat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)\right]$$

$$\hat{g} = -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} \qquad \hat{f} \downarrow \quad \uparrow \hat{\Delta}$$

Gap equation

$$\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) = N_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \left\langle V(\boldsymbol{k}_F, \boldsymbol{k}'_F) \hat{f}(\boldsymbol{k}'_F, \boldsymbol{r}, \omega_n) \right\rangle_{\boldsymbol{k}'_F}$$

pair potential : $V(\boldsymbol{k}_F, \boldsymbol{k}'_F) = 3g_1 \boldsymbol{k}_F \cdot \boldsymbol{k}'_F$

Mass current

$$\hat{g} = \begin{pmatrix} g_0 + g_z & g_x - ig_y \\ g_x + ig_y & g_0 - g_z \end{pmatrix}$$

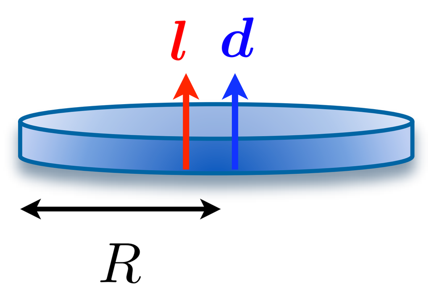
$$\boldsymbol{j}(\boldsymbol{r}) = mN_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)] \rangle_{\boldsymbol{k}_F}$$

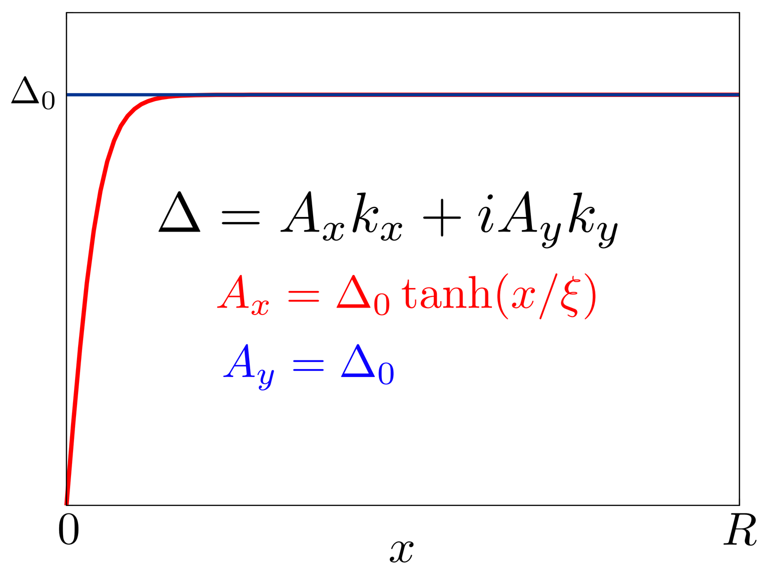
$$\boldsymbol{j}(\boldsymbol{r}, E) = \langle \boldsymbol{j}(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = mN_0 \langle \boldsymbol{v}_F \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

Local density of states (LDOS)

$$N(\boldsymbol{r}, E) = \langle N(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = N_0 \langle \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

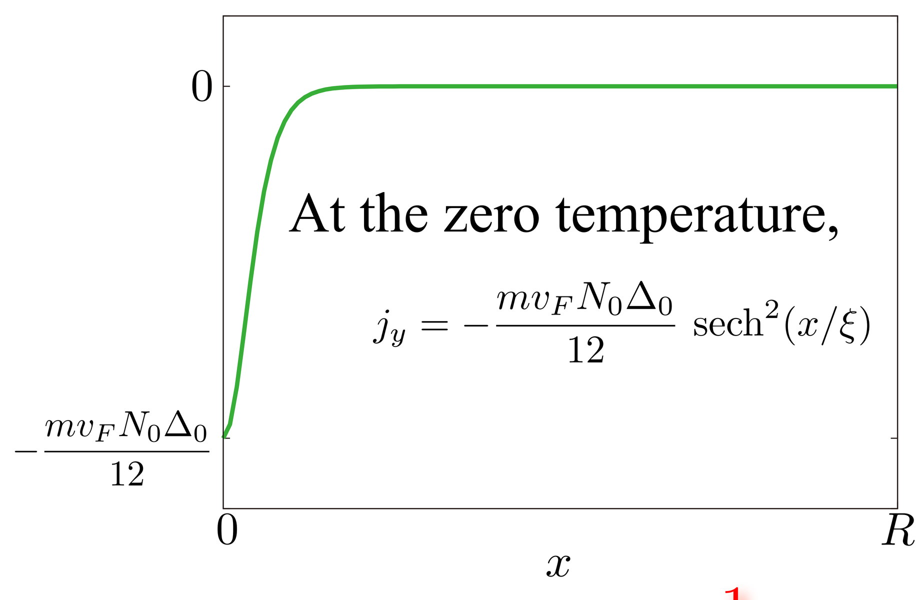
Result





$$g_0 = \frac{1}{\sqrt{\omega_n^2 + \Delta_0^2 \sin^2\theta}} \left[\omega_n + \frac{\Delta_0^2 \sin^2\theta \cos^2\phi}{2(\omega_n + i\Delta_0 \sin\theta \sin\phi)} \mathrm{sech}^2(x/\xi)\right]$$

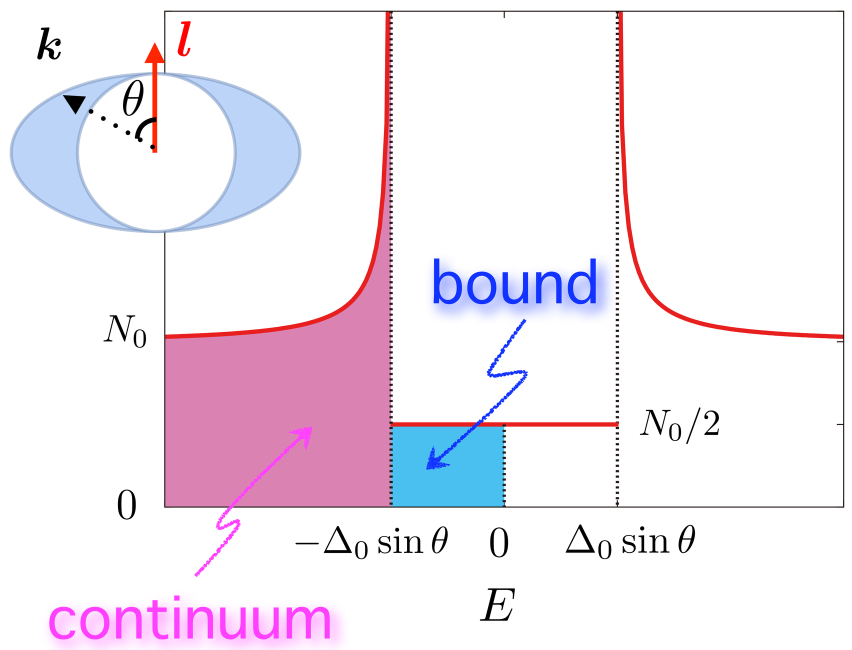
$(k_x = \cos\phi \sin\theta, k_y = \sin\phi \sin\theta)$



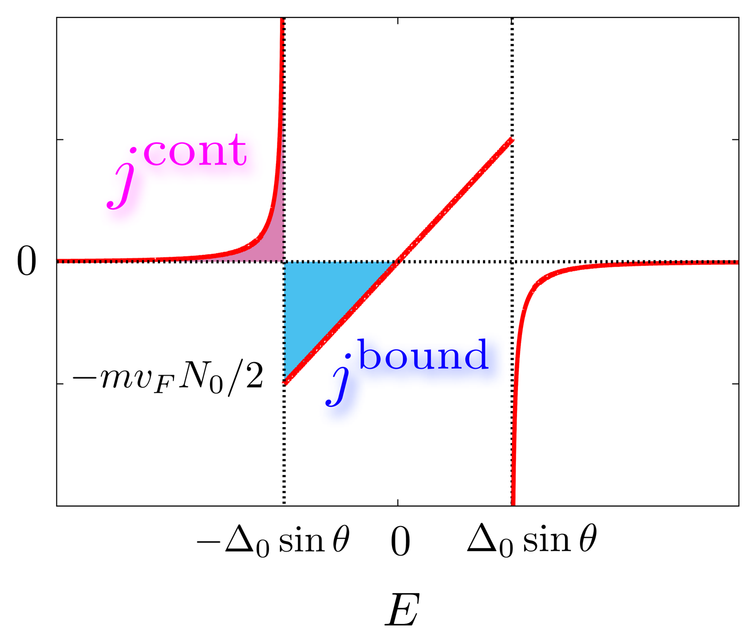
When $\xi \ll R$, $L_z = \frac{1}{2}N\hbar$

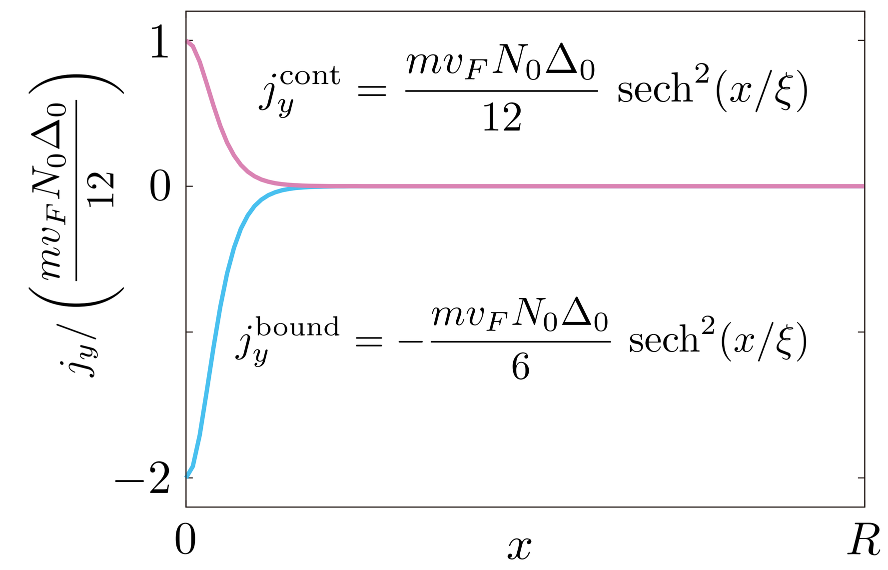
Bound state and continuum

$$N(\sin\theta, x = 0, E) = \int_0^{2\pi} \frac{d\phi}{2\pi} N(\boldsymbol{k}, x = 0, E)$$



$$j_y(\sin\theta, x = 0, E) = \int_0^{2\pi} \frac{d\phi}{2\pi} j_y(\boldsymbol{k}, x = 0, E)$$





$L_z^{\mathrm{bound}} = N\hbar$

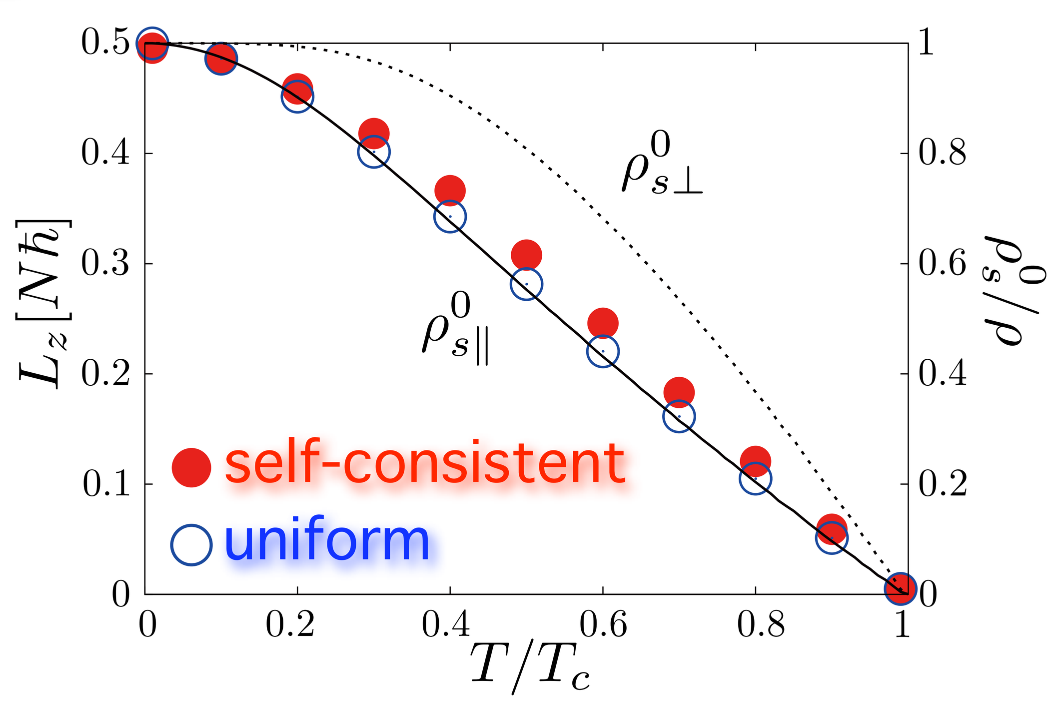
$L_z^{\mathrm{cont}} = -\frac{1}{2}N\hbar$

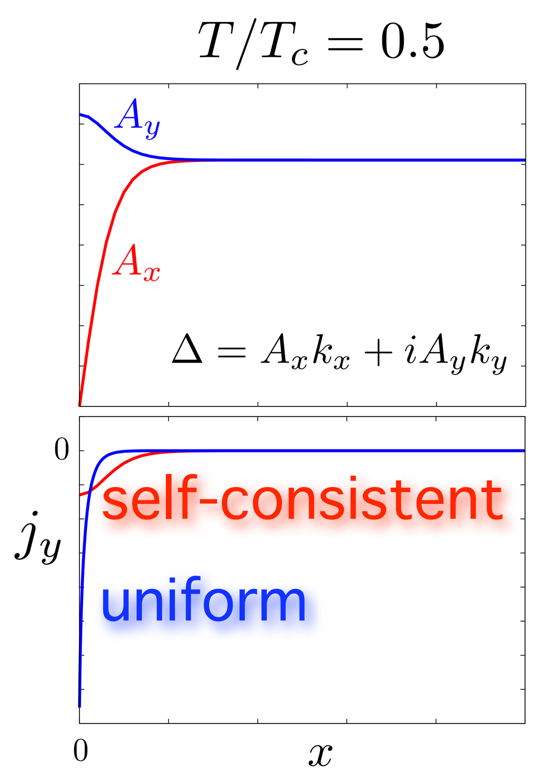
$\Rightarrow L_z = \frac{1}{2}N\hbar$

$$\boldsymbol{j}^{\mathrm{bound}}(x) = \left\langle \int_{-\Delta_0 \sin\theta}^{0} dE\, \boldsymbol{j}(\boldsymbol{k}, x, E) \right\rangle_{\boldsymbol{k}}$$

$$\boldsymbol{j}^{\mathrm{cont}}(x) = \left\langle \int_{-\infty}^{-\Delta_0 \sin\theta} dE\, \boldsymbol{j}(\boldsymbol{k}, x, E) \right\rangle_{\boldsymbol{k}}$$

Temperature dependence





$$\rho^0_{s\parallel} = 3\rho \left\langle k_z^2 \left(1 - \int_0^\infty \frac{d\omega}{2k_BT} \mathrm{sech}^2 \frac{\sqrt{\omega^2 + |\Delta(\boldsymbol{k})|^2}}{2k_BT}\right)\right\rangle_{\boldsymbol{k}} \qquad \rho^0_{s\perp} = 3\rho \left\langle k_x^2 \left(1 - \int_0^\infty \frac{d\omega}{2k_BT} \mathrm{sech}^2 \frac{\sqrt{\omega^2 + |\Delta(\boldsymbol{k})|^2}}{2k_BT}\right)\right\rangle_{\boldsymbol{k}}$$

M.C. Cross, JLTP **21**, 525 (1975).

Summary

- Half of the edge mass current from the bound state is canceled by that from the continuum state. Then, the magnitude of the total angular momentum is $L = N\hbar/2$.
- The angular momentum by the edge mass current is slightly larger than $\rho^0_{s\parallel}$ in the temperature region between $T = 0$ and T_c .

$\Rightarrow$ L^{bound} and L^{cont} in finite temperatures