PS1.15

Mass Current at a Domain Wall in Superfluid ^{3}He A-Phase

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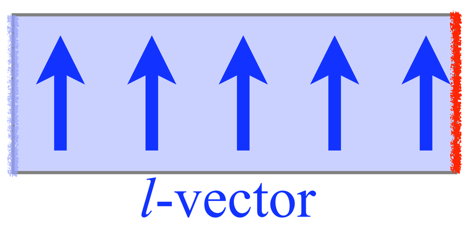
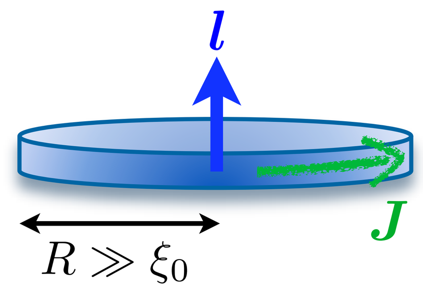
Edge Mass Current in A-Phase

$\boldsymbol{d} = \Delta_0 \hat{\boldsymbol{z}}(k_x + i k_y)$

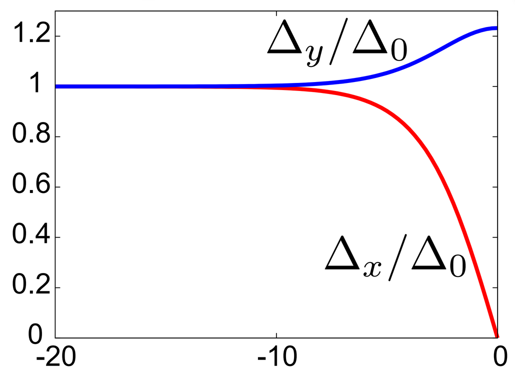




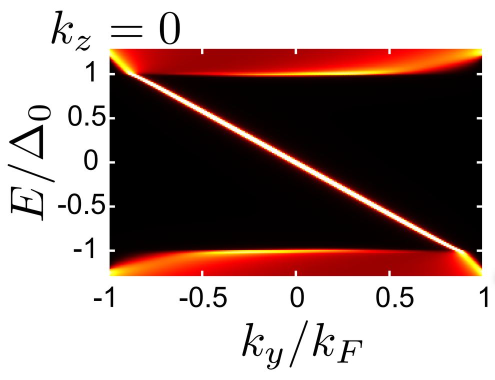
Topological number

$\nu = -1$ $\nu = 0$

- Order parameter



- Edge state

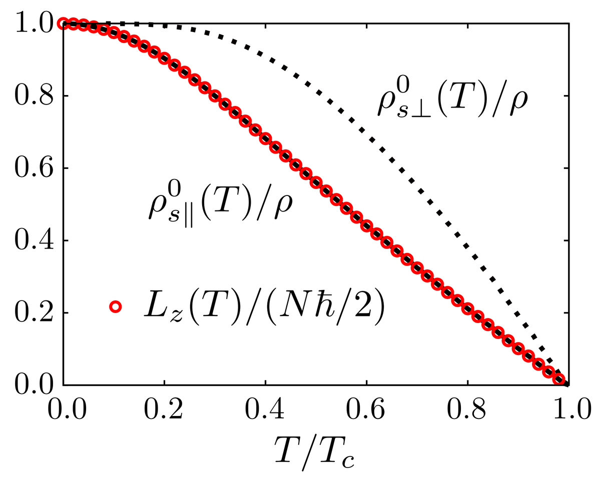


- Angular momentum

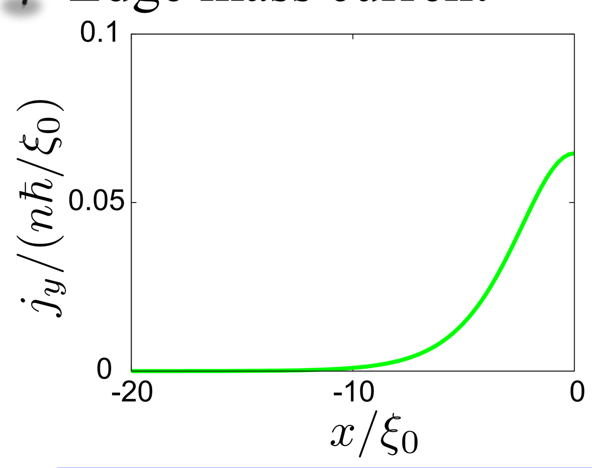
$L_z = \dfrac{N\hbar}{2}$

N : total number of ^{3}He atoms

- Temperature dependence



- Edge mass current



$J_y = \dfrac{n\hbar}{4}$

n : density of ^{3}He atoms

Y. Tsutsumi and K. Machida, PRB **85**, 100506(R) (2012).
J. A. Sauls, PRB **84**, 2145009 (2011).

- Edge mass current by topological phase transition seems to relate to intrinsic angular momentum.
- I have checked the relation by investigating mass current at a domain wall between different topological states.

Quasiclassical Theory

Eilenberger equation

$$\left(\omega_n + \hbar v_F \hat{k}_x \frac{\partial}{\partial x}\right) f = \Delta g, \quad \left(\omega_n - \hbar v_F \hat{k}_x \frac{\partial}{\partial x}\right) \underline{f} = \Delta^* g,$$

$$g^2 = 1 - f\underline{f}.$$

$g,\ f,\ \underline{f}$ ↓ ↑ Δ

Gap equation

$$\Delta(\hat{\boldsymbol{k}}, x) = N_0 \pi k_B T \sum_{0<\omega_n\le\omega_c} \left\langle V(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}') \left[f(\hat{\boldsymbol{k}}', x, \omega_n) + \underline{f}^*(\hat{\boldsymbol{k}}', x, \omega_n) \right] \right\rangle_{\hat{\boldsymbol{k}}'}.$$

Riccati equations

$$\hbar v_F \hat{k}_x \frac{\partial}{\partial x} a = \Delta - \Delta^* a^2 - 2\omega_n a, \quad -\hbar v_F \hat{k}_x \frac{\partial}{\partial x} b = \Delta^* - \Delta b^2 - 2\omega_n b.$$

$$a = f/(1+g), \quad b = \underline{f}/(1+g).$$

Initial values

$$a(\hat{\boldsymbol{k}}, -\infty, \omega_n) = \frac{\Delta(\hat{\boldsymbol{k}}, -\infty)}{\omega_n + \sqrt{\omega_n^2 + |\Delta(\hat{\boldsymbol{k}}, -\infty)|^2}}, \quad b(\hat{\boldsymbol{k}}, +\infty, \omega_n) = \frac{\Delta^*(\hat{\boldsymbol{k}}, +\infty)}{\omega_n + \sqrt{\omega_n^2 + |\Delta(\hat{\boldsymbol{k}}, +\infty)|^2}}.$$

Local density of states (LDOS)

$$N(\hat{\boldsymbol{k}}, x, E) = N_0 \,\mathrm{Re}\left[g(\hat{\boldsymbol{k}}, x, \omega_n)|_{i\omega_n \to E + i\eta} \right].$$

Mass current

$$\boldsymbol{j}(x) = N_0 \pi k_B T \sum_{|\omega_n|\le\omega_c} \left\langle m \boldsymbol{v}_F \,\mathrm{Im}\left[g(\hat{\boldsymbol{k}}, x, \omega_n) \right] \right\rangle_{\hat{\boldsymbol{k}}},$$

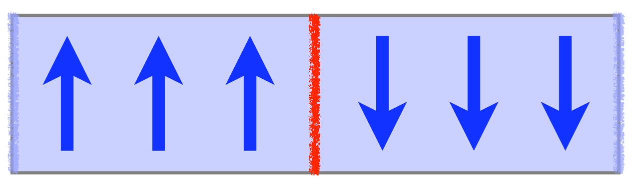
$$= \int_{-\infty}^{\infty} dE F(E) \left\langle m \boldsymbol{v}_F N(\hat{\boldsymbol{k}}, x, E) \right\rangle_{\hat{\boldsymbol{k}}}.$$

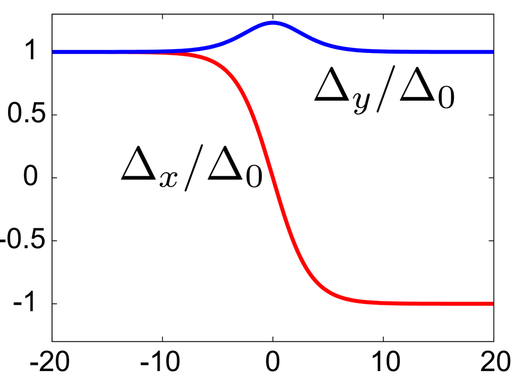
$F(E)$: Fermi distribution

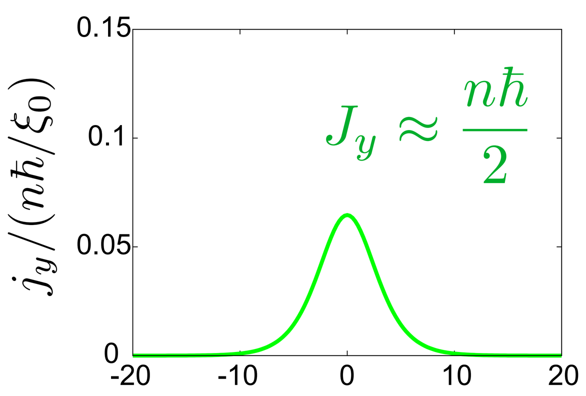
Mass Current at Domain Wall

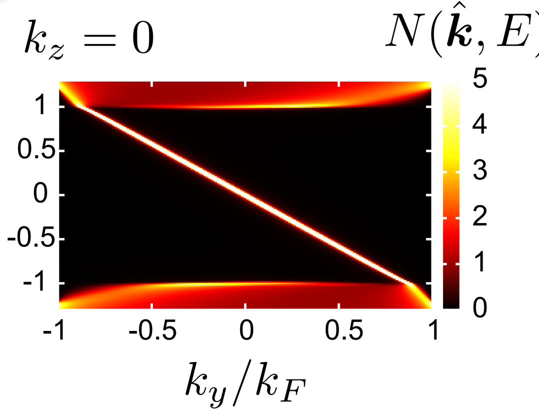
- System
- Order parameter
- Mass current
- Bound state

$\boldsymbol{d}_{\rm L} = \Delta_0 \hat{\boldsymbol{x}}(k_x + i k_y)$ $\boldsymbol{d}_{\rm R} = \Delta_0 \hat{\boldsymbol{x}}(-k_x + i k_y)$









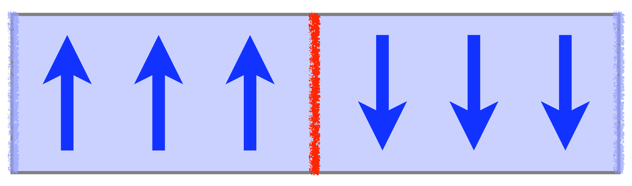
degenerate eigenstate for mirror operator $\mathcal{M}_x$

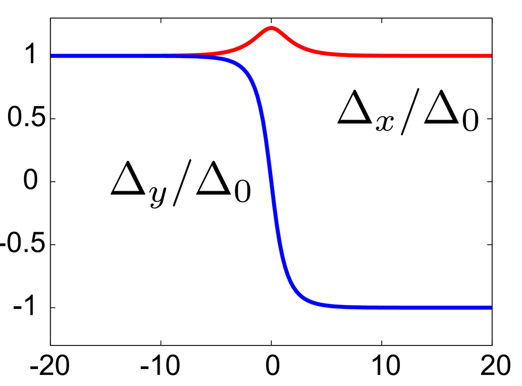
Each eigenstate has particle-hole symmetry.

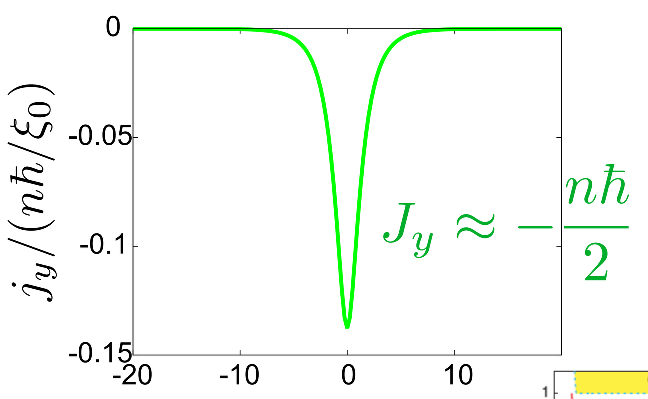
$\mathcal{C}\mathcal{M}_x\mathcal{C}^{-1} = \mathcal{M}_x$

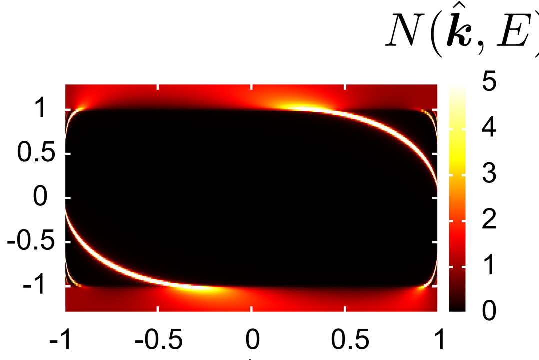
$(\boldsymbol{k}, E) \leftrightarrow (-\boldsymbol{k}, -E)$

$\boldsymbol{d}_{\rm L} = \Delta_0 \hat{\boldsymbol{x}}(k_x + i k_y)$ $\boldsymbol{d}_{\rm R} = \Delta_0 \hat{\boldsymbol{x}}(k_x - i k_y)$







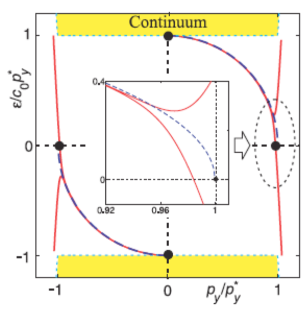


Particle-hole symmetry changes eigenstate for mirror operator.

$\mathcal{C}\mathcal{M}_x\mathcal{C}^{-1} = -\mathcal{M}_x$

$$\Delta J_y = \frac{n\hbar}{4} \sum_{E=0} \left(\frac{k_y}{k_F}\right)^2 \mathrm{sgn}\left[\frac{\partial E}{\partial k_y}\right] = -n\hbar$$

M. M. Salomaa and G. E. Volovik, JLTP **74**, 319 (1989).
G. E. Volovik, JETP Lett. **61**, 958 (1995).



M. A. Silaev and G. E. Volovik, PRB **86**, 214511 (2012)
by Bogoliubov-de Gennes theory

Topological number

$\nu_{\rm L} = -1$ $\nu_{\rm R} = +1$

$$\nu = 2(\nu_{\rm L} - \nu_{\rm R}) = \sum_{E=0} \mathrm{sgn}\left[\frac{\partial E}{\partial k_y}\right]$$

$$\mathcal{H}(\boldsymbol{k}) = \epsilon(\boldsymbol{k})\tau_3 + \Delta_x k_x \tau_1 + \Delta_y k_y \tau_2 \equiv \boldsymbol{m}(\boldsymbol{k}) \cdot \boldsymbol{\tau}$$

$$\epsilon(\boldsymbol{k}) = \frac{k_x^2 + k_y^2 + k_z^2 - k_F^2}{2m}$$

$$\nu(k_z) = \frac{1}{4\pi} \int d^2k \hat{\boldsymbol{m}} \cdot \left(\frac{\partial \hat{\boldsymbol{m}}}{\partial k_x} \times \frac{\partial \hat{\boldsymbol{m}}}{\partial k_y}\right) = -\,\mathrm{sgn}(\Delta_x \Delta_y), \ (|k_z| < k_F).$$

G. E. Volovik, *The Universe in a Helium Droplet* (2003).

Summary

- Even direction of mass current at domain wall depends on boundary condition.
- Topological mass current reflects the bulk state with special symmetry, separated eigenstate has particle-hole symmetry.

$$\mathcal{H}(\boldsymbol{k}, x) = \begin{pmatrix} \epsilon(\boldsymbol{k}) & \Delta(\boldsymbol{k}, x) \\ -\Delta^*(-\boldsymbol{k}, x) & -\epsilon(-\boldsymbol{k}) \end{pmatrix}$$

particle-hole symmetry: $\mathcal{C} = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

$$\mathcal{C}\mathcal{H}(\boldsymbol{k}, x)\mathcal{C}^{-1} = \begin{pmatrix} -\epsilon(-\boldsymbol{k}) & -\Delta^*(-\boldsymbol{k}, x) \\ \Delta(\boldsymbol{k}, x) & \epsilon(\boldsymbol{k}) \end{pmatrix} = -\mathcal{H}^*(-\boldsymbol{k}, x)$$

mirror symmetry: $\mathcal{M}_x = \begin{pmatrix} M_x & \\ & \pm M_x \end{pmatrix}$

$$M_x \epsilon(\boldsymbol{k}) M_x^{-1} = \epsilon(\boldsymbol{k})$$

$$M_x \Delta(\boldsymbol{k}, x) M_x^{-1} = \Delta_x(-x)(-k_x) + i\Delta_y(-x)k_y = \pm\Delta(\boldsymbol{k}, x)$$

$$[\mathcal{H}, \mathcal{M}_x] = 0$$

$$\mathcal{C}\mathcal{M}_x\mathcal{C}^{-1} = \pm\mathcal{M}_x$$

Cf. Y. Ueno *et al.*, arXiv:1303.0202.