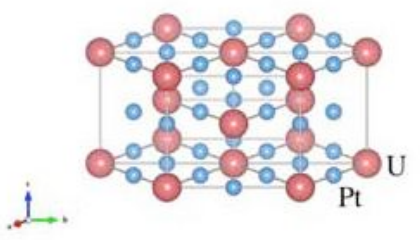
7P-110

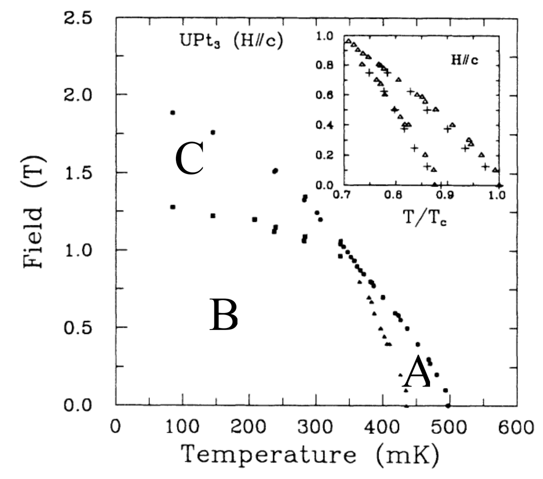
UPt$_3$ as a Topological Superconductor

Y. Tsutsumi[1], M. Ishikawa[2], T. Kawakami[3], T. Mizushima[2], M. Sato[4], M. Ichioka[2], and K. Machida[2]
[1]Condensed Matter Theory Laboratory, RIKEN, [2]Department of Physics, Okayama University,
[3]WPI-MANA, National Institute for Materials Science, [4]Department of Applied Physics, Nagoya University

Introduction



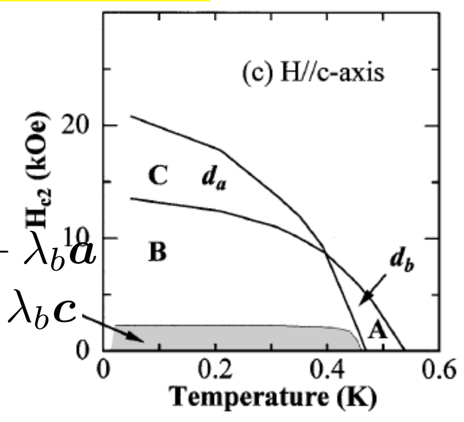
D_{6h}: hexagonal



S. Adenwalla *et al.*, PRL **65**, 2298 (1990).

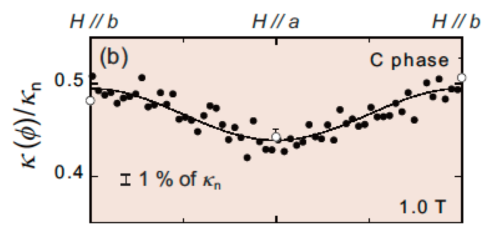
☑ line node and point node
☑ spin-triplet state

Knight shift

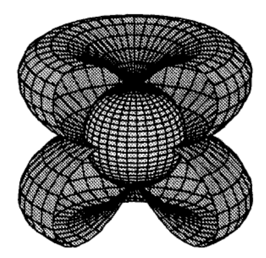


H. Tou *et al.*, PRL **80**, 3129 (1998).

thermal conductivity



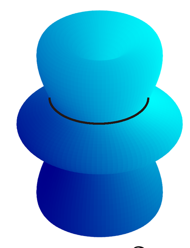
E$_{2u}$ representation



$\boldsymbol{d} \propto (k_a + ik_b)^2 k_c \boldsymbol{c}$

J. A. Sauls, Adv. Phys. **43**, 113 (1994).

E$_{1u}$ representation



$\boldsymbol{d} \propto \lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{a}$

$\boldsymbol{d} \propto \lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{c}$

$\lambda_a = k_a(5k_c^2 - k^2)$

$\lambda_b = k_b(5k_c^2 - k^2)$

Y. Machida *et al.*, PRL **108**, 157002 (2012).
Y. Tsutsumi *et al.*, JPSJ **81**, 074717 (2012).

- We have studied topological states (edge and vortex states) in the E$_{1u}$ representation with time-reversal and particle-hole symmetries.
- We suggest detection of the topological property distinguishable from the E$_{2u}$ representation.

Formulation

$\Delta/E_F \ll 1$

Eilenberger equation

$$-i\hbar \boldsymbol{v}_F(\bar{\boldsymbol{k}}) \cdot \boldsymbol{\nabla} \hat{g}(\bar{\boldsymbol{k}}, \boldsymbol{r}, \omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\bar{\boldsymbol{k}}, \boldsymbol{r}) \\ \hat{\Delta}^\dagger(\bar{\boldsymbol{k}}, \boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \hat{g}(\bar{\boldsymbol{k}}, \boldsymbol{r}, \omega_n) \right]$$

$$\hat{g} = -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} \qquad \hat{\Delta} \qquad \bar{\boldsymbol{k}} = \boldsymbol{k}/k_F$$

Gap equation

$$\hat{\Delta}(\bar{\boldsymbol{k}}, \boldsymbol{r}) = N_0 \pi k_B T \sum_{|\omega_n| \leq \omega_c} \left\langle V(\bar{\boldsymbol{k}}, \bar{\boldsymbol{k}}') \hat{f}(\bar{\boldsymbol{k}}', \boldsymbol{r}, \omega_n) \right\rangle_{\bar{\boldsymbol{k}}'}$$

Local density of states (LDOS)

$$\hat{g} = \begin{pmatrix} g_0 + g_z & g_x - ig_y \\ g_x + ig_y & g_0 - g_z \end{pmatrix}$$

$$N(\boldsymbol{r}, E) = \langle N(\bar{\boldsymbol{k}}, \boldsymbol{r}, E) \rangle_{\bar{\boldsymbol{k}}} = N_0 \left\langle \mathrm{Re}[g_0(\bar{\boldsymbol{k}}, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \right\rangle_{\bar{\boldsymbol{k}}}$$

vortex // c

Bogoliubov-de Gennes equation

$$\int d\boldsymbol{\rho}_2 \begin{pmatrix} \hat{\epsilon}_{k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) & \hat{\Delta}_{k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \\ -\hat{\Delta}^\dagger_{-k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) & -\hat{\epsilon}^{\mathrm{T}}_{-k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \end{pmatrix} u_{\nu,k_c}(\boldsymbol{\rho}_2) = E_{\nu,k_c} u_{\nu,k_c}(\boldsymbol{\rho}_1)$$

$$\boldsymbol{\rho} = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2, \quad \boldsymbol{\rho}_{12} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$$

$$\hat{\epsilon}_{k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \delta(\boldsymbol{\rho}_{12}) \left[-\frac{\hbar^2 \nabla^2_{\boldsymbol{\rho}}}{2m} - \frac{\hbar^2}{2m}(k_F^2 - k_c^2) \right] \hat{1}$$

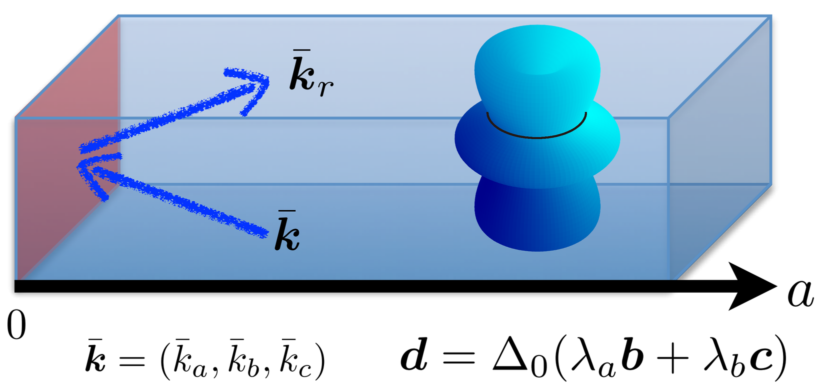
$$\hat{\Delta}_{k_c}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \int \frac{d\boldsymbol{k}^{2D}}{(2\pi)^2} \hat{\Delta}(\boldsymbol{k}, \boldsymbol{\rho}) e^{i\boldsymbol{k}^{2D} \cdot \boldsymbol{\rho}_{12}}$$

Edge State

System

specular edge

$\hat{g}(\bar{\boldsymbol{k}}, a=0, \omega_n) = \hat{g}(\bar{\boldsymbol{k}}_r, a=0, \omega_n)$

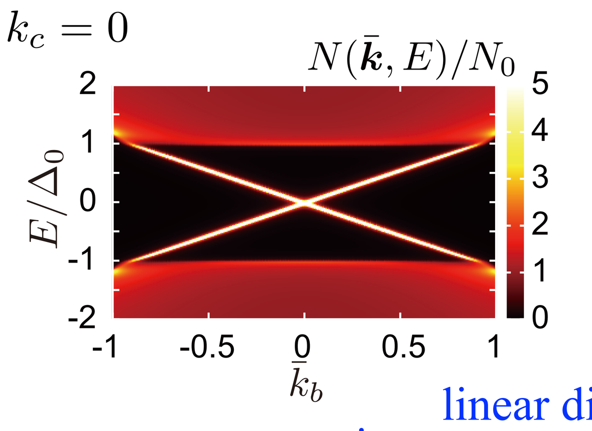


$\bar{\boldsymbol{k}} = (\bar{k}_a, \bar{k}_b, \bar{k}_c) \qquad \boldsymbol{d} = \Delta_0(\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{c})$

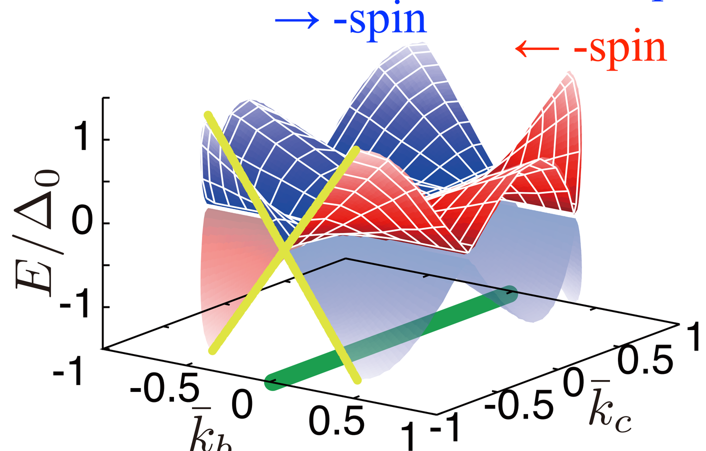
$\bar{\boldsymbol{k}}_r = (-\bar{k}_a, \bar{k}_b, \bar{k}_c)$

Energy dispersion

$k_c = 0$



linear dispersion



$E_0 = \pm\Delta_0 |5\bar{k}_c^2 - 1| \bar{k}_b$ +: ← -spin, -: → -spin

Majorana Ising anisotropy

General Hamiltonian

$$\hat{\mathcal{H}}(\boldsymbol{k}) = \hat{\mathcal{H}}_0(\boldsymbol{k}) + \hat{\mathcal{H}}_{\boldsymbol{H}}$$

$$\hat{\mathcal{H}}_0(\boldsymbol{k}) = \begin{pmatrix} \hat{\epsilon}(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}) & -\hat{\epsilon}^{\mathrm{T}}(-\boldsymbol{k}) \end{pmatrix}, \quad \hat{\mathcal{H}}_{\boldsymbol{H}} = \begin{pmatrix} -\mu_B \boldsymbol{H} \cdot \hat{\boldsymbol{\sigma}} & 0 \\ 0 & \mu_B \boldsymbol{H} \cdot \hat{\boldsymbol{\sigma}}^{\mathrm{T}} \end{pmatrix}$$

$\hat{\epsilon}(\boldsymbol{k})$: general normal state Hamiltonian with D_{6h} symmetry

Mirror reflection in *ca*-plane

$$\hat{\mathcal{M}}_{ca} = \begin{pmatrix} \hat{M}_{ca} & \\ & \hat{M}^*_{ca} \end{pmatrix}, \quad \hat{M}_{ca} \propto i\hat{\sigma}_b$$

$$\hat{\mathcal{M}}_{ca} \hat{\mathcal{H}}_0(k_a, k_b, k_c) \hat{\mathcal{M}}^\dagger_{ca} = \hat{\mathcal{H}}_0(k_a, -k_b, k_c)$$

$$\left[\hat{\mathcal{M}}_{ca}, \hat{\mathcal{H}}_0\right] = 0, \quad (k_b = 0)$$

Mirror chiral operator

$$\Gamma = \mathcal{TC}\hat{\mathcal{M}}_{ca}$$

$$\left\{\Gamma, \hat{\mathcal{H}}_0\right\} = 0, \quad (k_b = 0)$$

1D winding number

$$w(k_c) = -\frac{1}{4\pi i} \int_{-\pi}^{\pi} dk_a \mathrm{tr}[\Gamma \hat{\mathcal{H}}_0^{-1} \partial_{k_a} \hat{\mathcal{H}}_0]$$

$$|w(k_c)| = 2, \quad (k_b = 0, \ |k_c| < k_F)$$

zero energy state

Magnetic field

$$\Gamma(\boldsymbol{H} \cdot \hat{\boldsymbol{\sigma}})\tau_z \Gamma^\dagger = (H_a \sigma_a - H_b \sigma_b + H_c \sigma_c)\tau_z$$

$w(k_c)$ is invariant without H_b

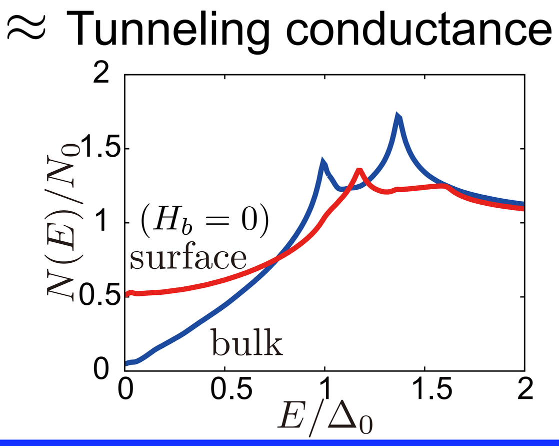
Zero energy state is protected by mirror chiral symmetry without H_b.

Majorana Ising anisotropy

$$E = \pm\sqrt{E_0^2 + (\mu_B H_b)^2}$$

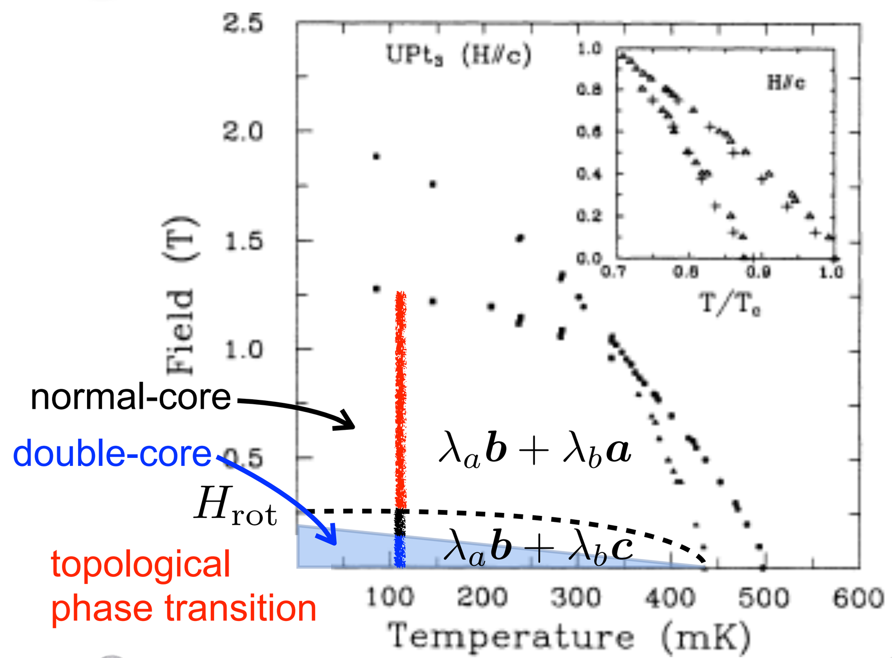
Surface LDOS

≈ Tunneling conductance



Vortex State

Phase diagram

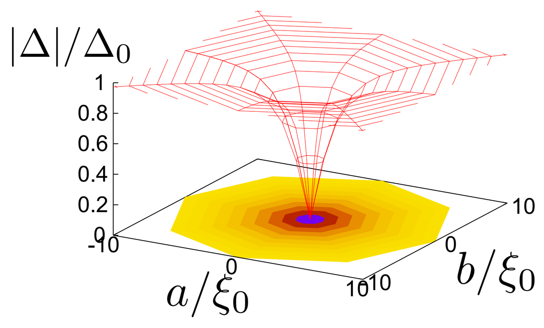


normal-core vortex
class D + mirror symmetry
Degenerate zero energy modes are topologically protected Majorana zero modes.

normal-core vortex
class D in 1D: $\mathbb{Z}_2$
Degenerate zero energy modes are topologically trivial.

double-core vortex
without zero energy mode

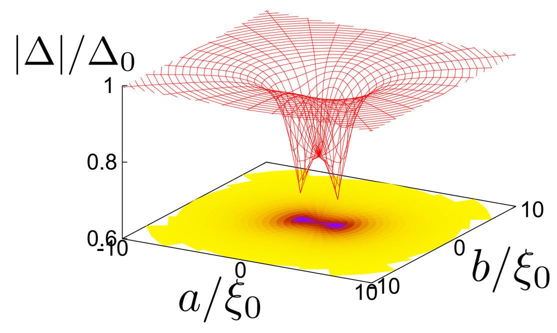
Normal-core vortex



$H > H_{\rm rot}$: $\boldsymbol{d} = \Delta_0 e^{i\varphi}(\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{a})$

$H < H_{\rm rot}$: $\boldsymbol{d} = \Delta_0 e^{i\varphi}(\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{c})$ $\quad r \to \infty$

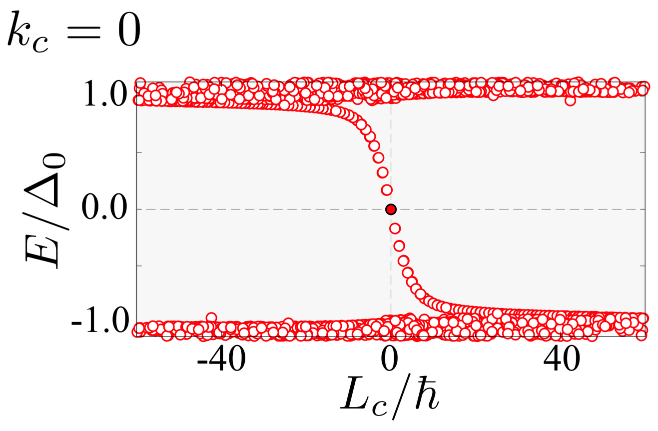
Double-core vortex

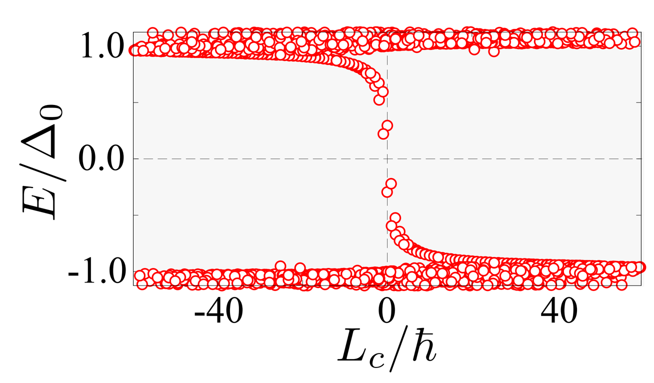


$\boldsymbol{d} = \Delta_0 e^{i\varphi}(\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{c}) \quad r \to \infty$

$\boldsymbol{d} = \Delta_{\rm core} \lambda_b \boldsymbol{a} \quad r = 0$

$k_c = 0$





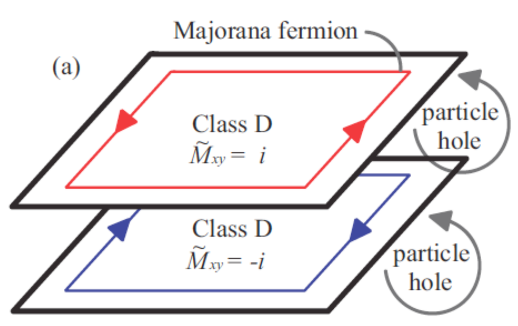
Symmetry protection

Mirror reflection in *ab*-plane

$$\hat{\mathcal{M}}^{(-)}_{ab} = \begin{pmatrix} \hat{M}_{ab} & \\ & -\hat{M}^*_{ab} \end{pmatrix}, \quad \hat{M}_{ab} \propto i\hat{\sigma}_c$$

$\hat{M}_{ab} \hat{\Delta}(\boldsymbol{k}) \hat{M}^{\mathrm{T}}_{ab} = -\hat{\Delta}(k_a, k_b, -k_c)$ in high field: $\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{a}$

$\left[\hat{\mathcal{M}}^{(-)}_{ab}, \hat{\mathcal{H}}(\boldsymbol{k})\right] = 0, \quad (k_c = 0)$ ~~$\lambda_a \boldsymbol{b} + \lambda_b \boldsymbol{c}$~~



Ueno *et al.*, arXiv:1303.0202.

Summary

We have studied topological states of UPt$_3$ B-phase.

- Edge state has linear dispersion with zero energy state showing Majorana Ising anisotropy, which will be detected by the tunneling spectroscopy.
- Vortex state undergoes topological phase transition from topologically trivial Dirac modes to topologically protected Majorana zero modes.

Y. Tsutsumi *et al.*, arXiv:1307.1264.