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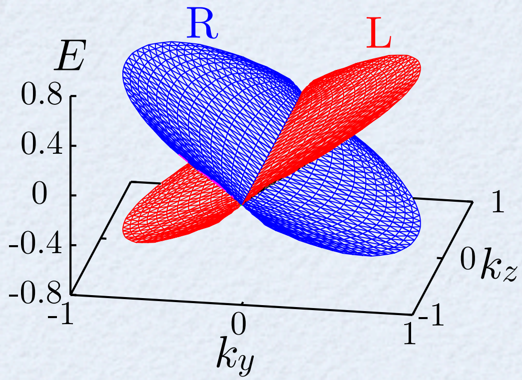
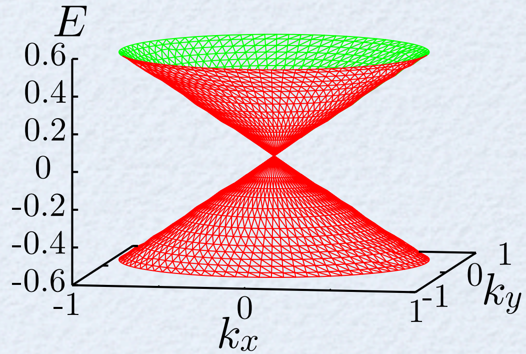
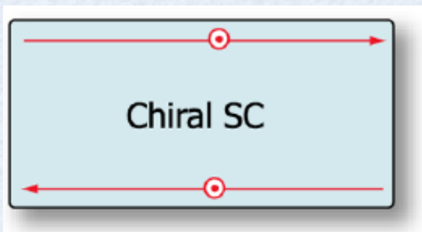
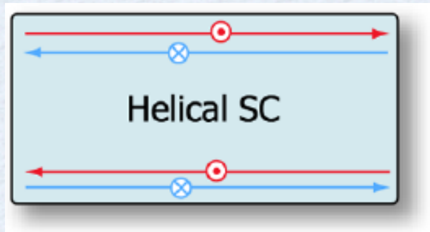
超流動ヘリウム３の
エッジ状態とエッジカレント

岡山大学自然科学研究科
堤康雅，水島健，市岡優典，町田一成

Edge state for A- and B-phases

	A-phase	B-phase
Order parameter	$(k_x + ik_y)d_z$	$k_x d_x + k_y d_y + k_z d_z$
Time reversal symmetry	broken	unbroken
Andreev bound state (Majorana fermion)		
Edge current		
	mass current	spin current

目的

LDOS, edge current (大きさ, 温度変化) に注目して,
A相とB相の性質の違いがどのように反映されるかを明らかにする.
Intrinsic angular momentum.

Quasi-classical theory

$\Delta/E_F \ll 1$ superfluid ^{3}He: $\sim 10^{-3}$

Eilenberger方程式

$$-i\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \left[\begin{pmatrix} i\omega_n \hat{1} & -\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F, \boldsymbol{r}) & -i\omega_n \hat{1} \end{pmatrix}, \widehat{g}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) \right]$$

Matsubara frequency: $\omega_n = (2n+1)\pi k_B T$

$$\widehat{g} = -i\pi \begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} = -i\pi \begin{pmatrix} (\hat{1}+\hat{a}\hat{b})^{-1} & 0 \\ 0 & (\hat{1}+\hat{b}\hat{a})^{-1} \end{pmatrix} \begin{pmatrix} \hat{1}-\hat{a}\hat{b} & 2i\hat{a} \\ -2i\hat{b} & -(\hat{1}-\hat{b}\hat{a}) \end{pmatrix}$$

Riccati方程式

$$\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta} - \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)\hat{\Delta}^\dagger \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

$$-\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta}^\dagger - \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)\hat{\Delta} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

オーダーパラメーターのエッジでの空間変化を仮定して,
Riccati方程式を解析的に解く.

Edge current and LDOS

$$\hat{g} = \begin{pmatrix} g_0 + g_z & g_x - ig_y \\ g_x + ig_y & g_0 - g_z \end{pmatrix}$$

Temperature dependence

mass current: $$\boldsymbol{j}(\boldsymbol{r}, T) = mN_0\pi k_B T \sum_{\omega_n} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)] \rangle_{\boldsymbol{k}_F}$$

spin current: $$\boldsymbol{j}_s^{\mu}(\boldsymbol{r}, T) = \frac{\hbar}{2} N_0\pi k_B T \sum_{\omega_n} \langle \boldsymbol{v}_F \mathrm{Im}[g_{\mu}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)] \rangle_{\boldsymbol{k}_F}$$

Energy spectrum

mass current:

$$\boldsymbol{j}(\boldsymbol{r}, E) = \langle \boldsymbol{j}(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = mN_0 \langle \boldsymbol{v}_F \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

spin current:

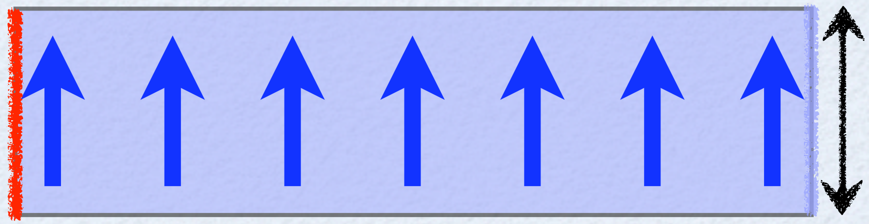
$$\boldsymbol{j}_s^{\mu}(\boldsymbol{r}, E) = \langle \boldsymbol{j}_s^{\mu}(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = \frac{\hbar}{2} N_0 \langle \boldsymbol{v}_F \mathrm{Re}[g_{\mu}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

local density of states (LDOS):

$$N(\boldsymbol{r}, E) = \langle N(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = N_0 \langle \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

System for A-phase

l-vector

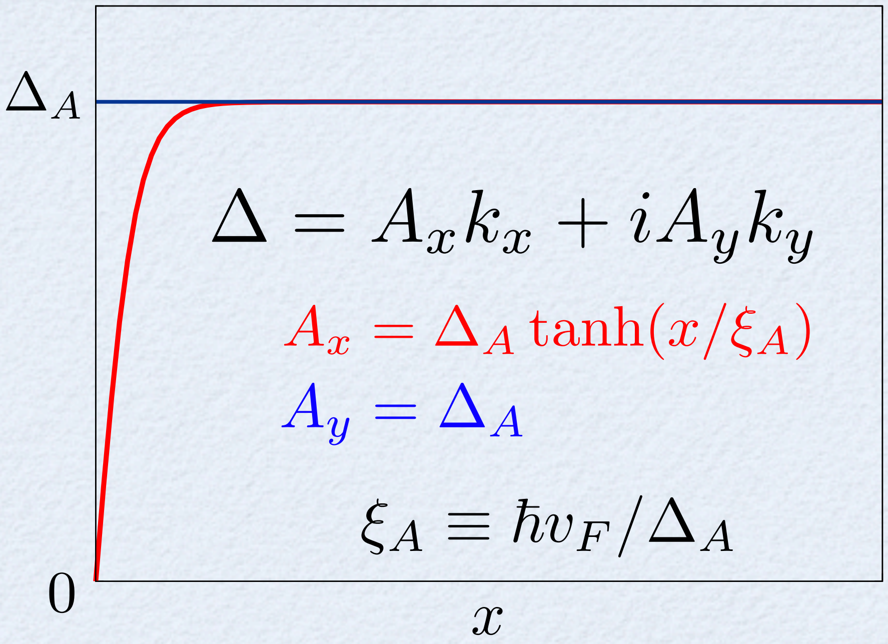


$\lesssim 10\xi \sim 1\mu\mathrm{m}$

specular edge

$x = 0$

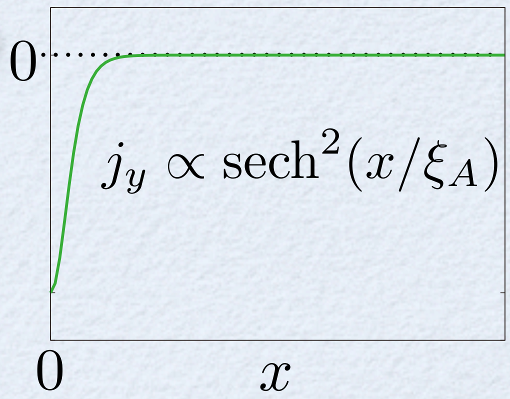
Order parameter



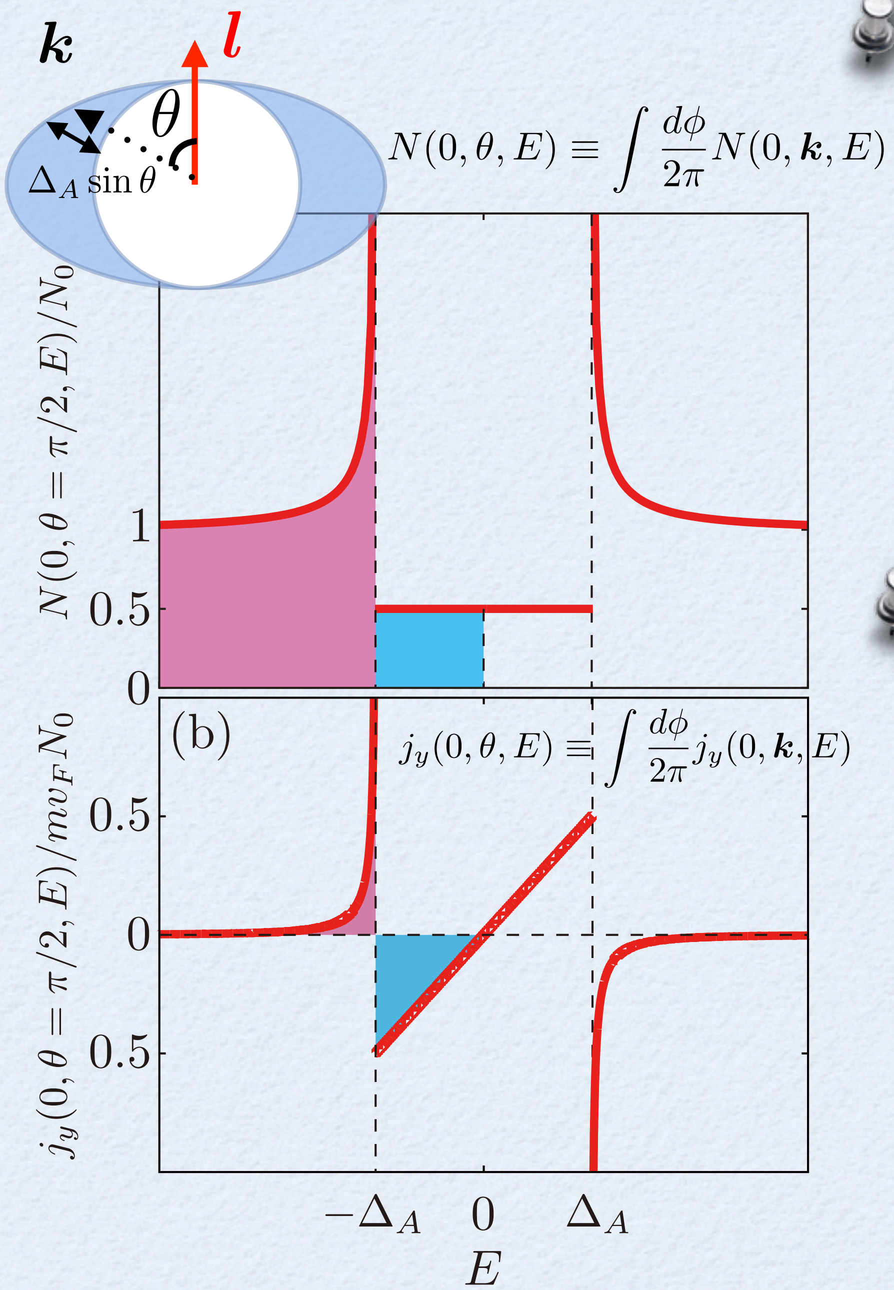
analytic solution: $(k_x = \cos\phi\sin\theta, k_y = \sin\phi\sin\theta)$

$$g_0 = \frac{1}{\sqrt{\omega_n^2 + \Delta_A^2 \sin^2\theta}} \left[\underbrace{\omega_n}_{\text{bulk}} + \underbrace{\frac{\Delta_A^2 \sin^2\theta \cos^2\phi}{2(\omega_n + i\Delta_A \sin\theta\sin\phi)} \operatorname{sech}^2\left(\frac{x}{\xi_A}\right)}_{\text{edge state}} \right]$$

Edge mass current



LDOS and edge mass current



LDOS

bound state: $|E| < \Delta_A \sin\theta$

$$N(0,\theta,E) = \frac{N_0}{2}$$ constant

continuum state: $|E| > \Delta_A \sin\theta$

$$N(0,\theta,E) = \frac{N_0}{2}\left(\underbrace{\frac{|E|}{\sqrt{E^2-\Delta_A^2\sin^2\theta}}}_{\text{bulk}} + \underbrace{1}_{\text{edge state}}\right)$$

Edge mass current

bound state:

$$j_y(0,\theta,E) = \frac{mv_F N_0}{2}\frac{E}{\Delta_A}$$ E-linear

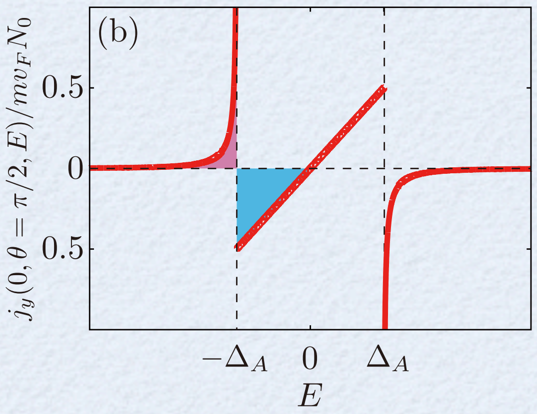
continuum state:

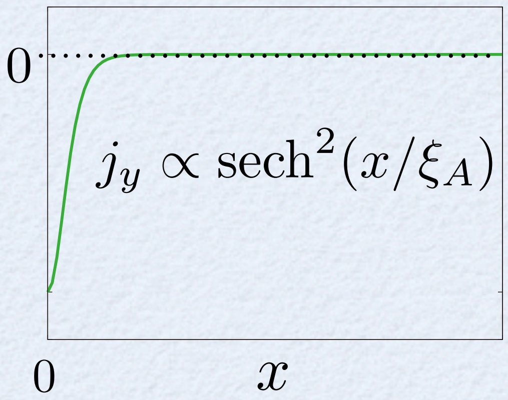
$$j_y(0,\theta,E) = -\frac{mv_F N_0}{4}\left(\mathrm{sgn}(E)\frac{\sqrt{E^2-\Delta_A^2\sin^2\theta}}{\Delta_A} + \frac{E|E|}{\Delta_A\sqrt{E^2-\Delta_A^2\sin^2\theta}} - 2\frac{E}{\Delta_A}\right)$$

$$\approx -\frac{mv_F N_0}{16}\sin\theta\left(\frac{\Delta_A\sin\theta}{E}\right)^3 \qquad (|E| \gg \Delta_A\sin\theta)$$

ギャップから離れるとE^{-3}で減衰.
マスカレントを運ぶのは,
ギャップ近傍の準粒子だけではない.

Total current and angular momentum





Zero temperature

Total mass current

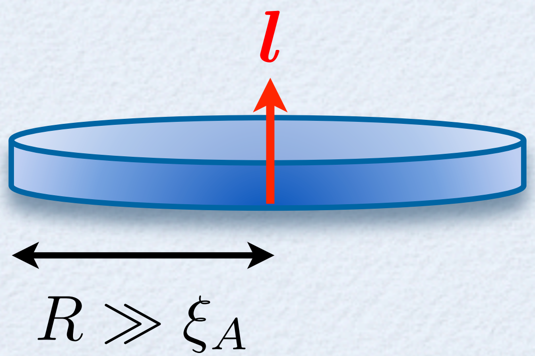
$$J_y^{\mathrm{bound}} = \int_0^\infty dx \left\langle \int_{-\Delta_A \sin\theta}^{0} dE\, j_y(x,\boldsymbol{k},E) \right\rangle_{\boldsymbol{k}} = -\frac{n\hbar}{2}$$

$$J_y^{\mathrm{cont}} = \int_0^\infty dx \left\langle \int_{-\infty}^{-\Delta_A \sin\theta} dE\, j_y(x,\boldsymbol{k},E) \right\rangle_{\boldsymbol{k}} = \frac{n\hbar}{4}$$

$$\left(N_0 = \frac{3}{mv_F^2} n \right)$$

$$J_y = J_y^{\mathrm{bound}} + J_y^{\mathrm{cont}} = -\frac{n\hbar}{4}$$

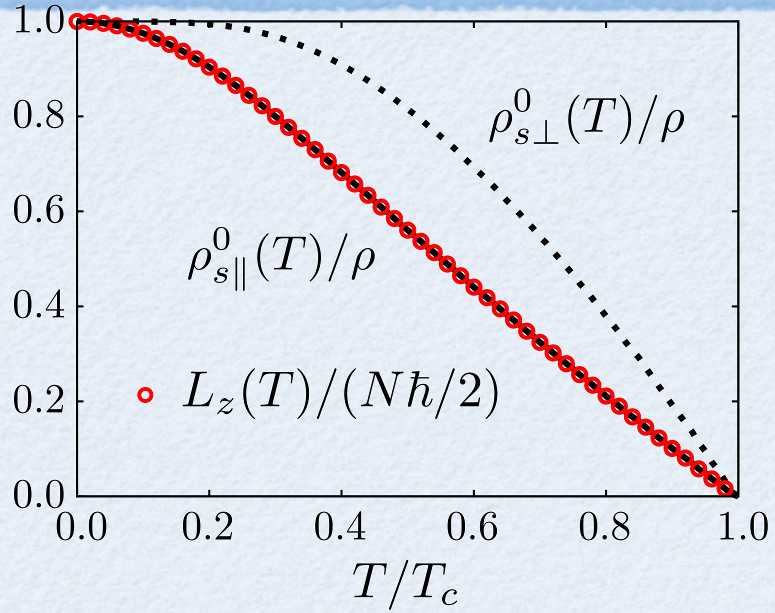
Angular momentum



$$L_z = \frac{N\hbar}{2}$$

(N : number of ^{3}He atoms in slab)

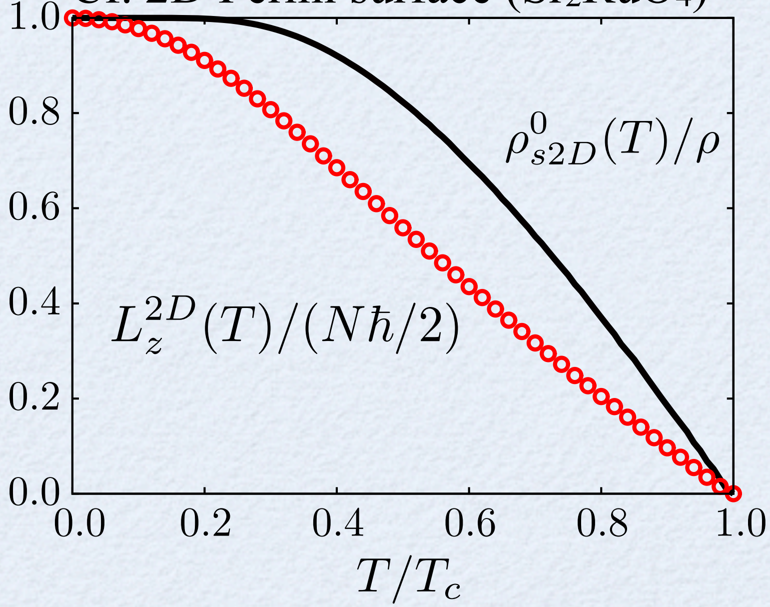
Temperature dependence



$$L_z(T)/(N\hbar/2) = \rho^0_{s\parallel}(T)/\rho$$

偶然？

Cf. 2D Fermi surface (Sr_2RuO_4)



$$L_z(T) = \frac{3}{4}N\hbar\frac{\pi k_B T}{\Delta_A}\sum_{\omega_n}\left[\frac{3\omega_n^2+\Delta_A^2}{\Delta_A^2}\sin^{-1}\frac{\Delta_A}{\sqrt{\omega_n^2+\Delta_A^2}} - 3\frac{|\omega_n|}{\Delta_A}\right]$$

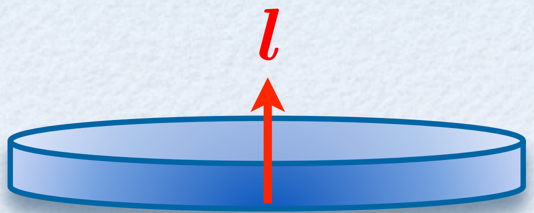
$$= \frac{N\hbar}{2}\left[1 - \underbrace{\left(\frac{\pi k_B T}{\Delta_A}\right)^2}_{\text{bound state}} + \underbrace{\mathrm{O}\left(\frac{\pi k_B T}{\Delta_A}\right)^4}_{\text{continuum state}}\right] \quad (\pi k_B T \ll \Delta_A)$$

bound state
(*E*-linear)

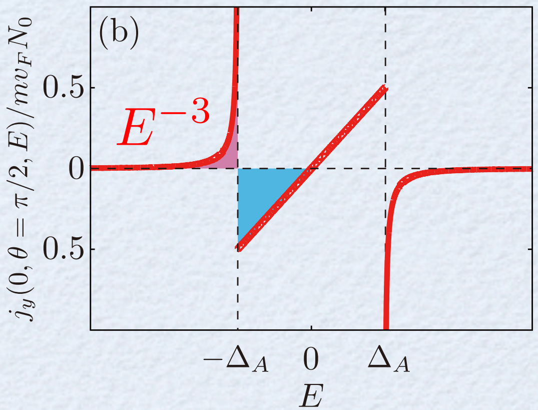
continuum state
(point node)

Y. Nagato, S. Higashitani, and K. Nagai, JPSJ **80**, 113706 (2011).
J.A. Sauls, PRB **84**, 214509 (2011).

Intrinsic angular momentum



$$L_z = \frac{N\hbar}{2}$$



Intrinsic angular momentum problem

クーパー対の角運動量の和が
全角運動量として現れるのか？

N個のヘリウム３原子全てが
クーパー対を組み，
それぞれが$\hbar$の角運動量を運ぶ．

角運動量は超流動密度と同様の
温度変化をする．

本研究の結果

エッジカレントを運ぶのはギャップ
近傍の準粒子だけではないが，
全準粒子が運んでいるわけではない．
Nは状態密度$N_0 = \frac{3}{mv_F^2}n$を通して現れる．

$$L_z(T)/(N\hbar/2) = \rho^0_{s\|}(T)/\rho$$

但し，偶然の一致の可能性あり．
低温での変化は表面束縛状態の
ギャップレス励起による．
2Dフェルミ面の場合は異なる温度変化．

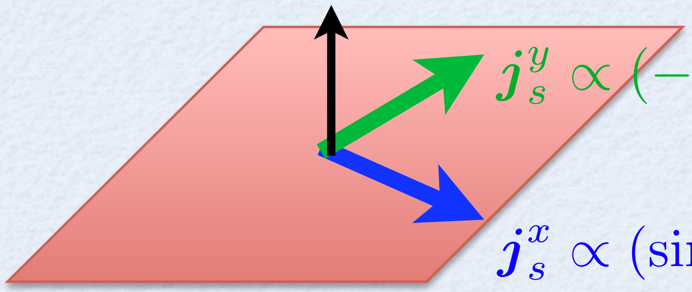
B-phase

System

$$\boldsymbol{\Delta} = \Delta_B R(\boldsymbol{z}, \theta_L)(k_x \boldsymbol{x} + k_y \boldsymbol{y} + k_z \boldsymbol{z})$$

n-vector

$$\theta_L = \cos^{-1}(-1/4) \approx 104^\circ$$



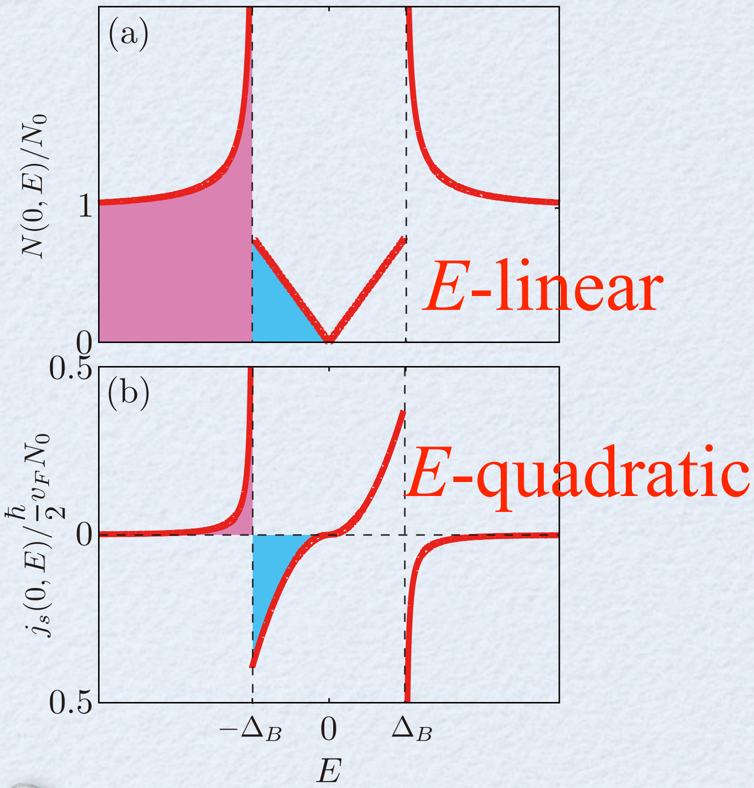
$$\boldsymbol{j}_s^y \propto (-\cos\theta_L \boldsymbol{x} + \sin\theta_L \boldsymbol{y})$$

$$\boldsymbol{j}_s^x \propto (\sin\theta_L \boldsymbol{x} + \cos\theta_L \boldsymbol{y})$$

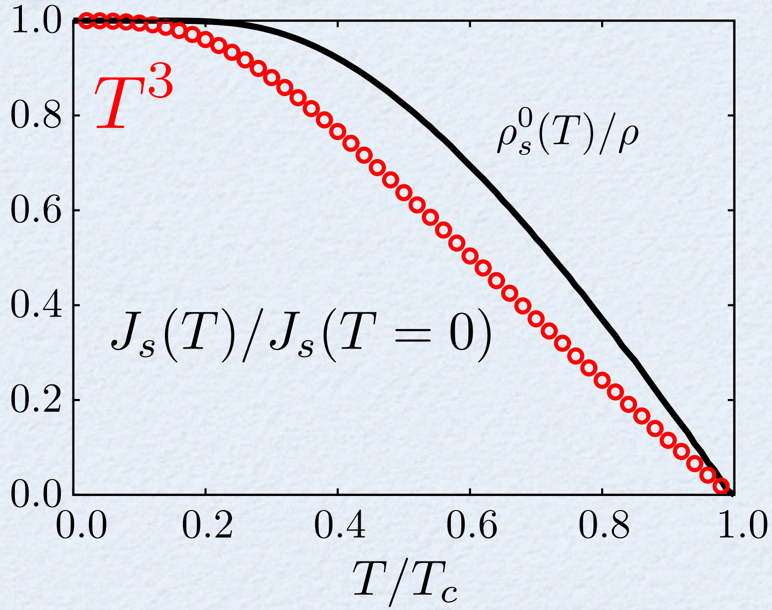
specular edge

$z = 0$

LDOS and edge spin current



Temperature dependence



Total spin current

$$|J_s| = \frac{\hbar}{2m}\frac{n\hbar}{6}$$

Cf. mass current in A-phase

$$|J| = \frac{n\hbar}{4}$$

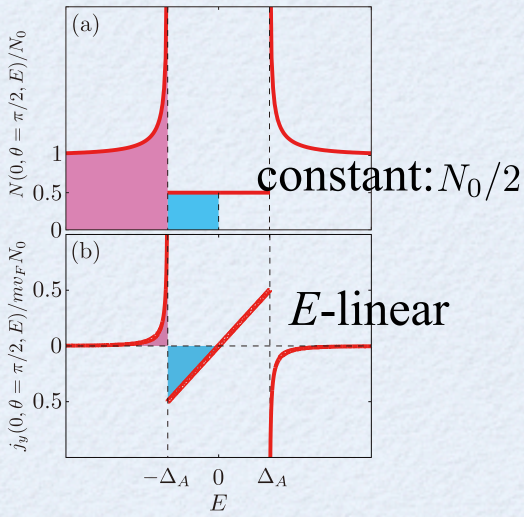
Summary

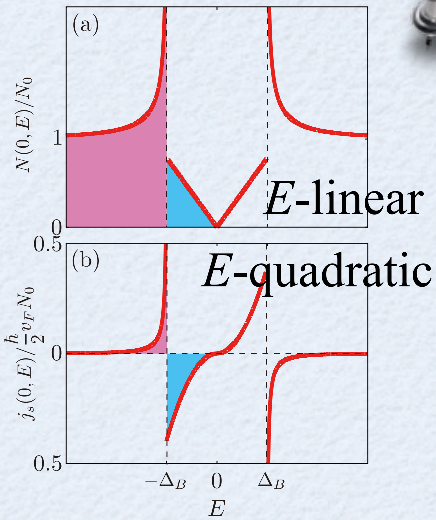
A相とB相の性質の違いがエッジ状態にどのように反映されるかを準古典理論に基づくRiccati方程式を解析的に解くことで研究した.

A-phase

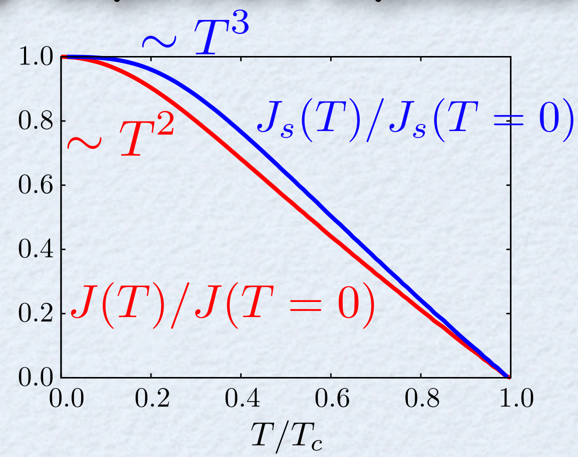
B-phase

- LDOS
- Edge current





- Temperature dependence



- Total current

$$|J| = \frac{n\hbar}{4}$$

$$|J_s| = \frac{\hbar}{2m}\frac{n\hbar}{6}$$

- Angular momentum

$$L_z = \frac{N\hbar}{2}$$

クーパー対の角運動量が直接現れているのではなさそう.

Y. Tsutsumi and K. Machida, PRB **85**, 100506(R) (2012).
Y. Tsutsumi and K. Machida, arXiv:1203.2722.