Covariant Gauges in String Field Theory

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"New covariant gauges in string field theory" PTP 117 (2007) 569, "Level truncated tachyon potential in various gauges" JHEP 0701 (2007) 028, and the work under study.

Introduction

Motivation and what we do

SFT has a huge gauge symmetry. \rightarrow Needs gauge-fixing

Siegel gauge has been a unique choice of covariant gauge since 1984 (even before the gauge-inv. action was found.)

In ordinary gauge theory (e.g. Abelian case)

$$S_{\text{gauge}} = \int d^D x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + B \partial_\mu A^\mu + \frac{\alpha}{2} B^2 + i \overline{c} \, \partial_\mu \partial^\mu c \right]$$

 α is a gauge parameter

 $\alpha = 1$ Feynman gauge $\alpha = 0$ Landau gauge

Siegel gauge gives Feynman gauge for massless gauge mode. Then what about Landau and other gauges? Tachyon potential in Siegel gauge showed singular behavior.



Alternative gauges have been desired

- To distinguish the physical and gauge artifact (or truncation-scheme artifact,)
- To obtain reliable results for wide range (Must for quantum analysis.)

In this talk,

Proposal of new covariant gauge

A single parameter family of gauges which naturally corresponds to the covariant gauges in ordinary gauge theory.

Application to tachyon potential

The identification of physical branch. Obtaining smooth potential.

SFT and Siegel gauge

Notations and Fundamentals

String field

$$\Phi_{1}[X^{\mu}(\sigma), c(\sigma), b(\sigma)] = \int d^{26}x \sum_{s} |s, x\rangle \psi_{s}(x)$$

= $\int \frac{d^{26}p}{(2\pi)^{26}} \sum_{s} |s, p\rangle \tilde{\psi}_{s}(p)$
= $\int \frac{d^{26}p}{(2\pi)^{26}} \left[\phi(p) + A_{\mu}(p) \alpha_{-1}^{\mu} + i\chi(p) c_{0} b_{-1} + \cdots \right] |0, p\rangle$

$$X^{\mu}(\sigma) = x^{\mu} + \sum_{n \neq 0} \frac{\alpha_n^{\mu}}{n} \cos(n\sigma) \text{ string coordinate } (gh\# = 0)$$

$$c(\sigma) = \sum_n c_n e^{-in\sigma} \text{ worldsheet ghost } (gh\# = 1)$$

$$b(\sigma) = \sum_n b_n e^{-in\sigma} \text{ worldsheet anti-ghost } (gh\# = -1)$$

$$[x^{\mu}, p^{\nu}] = i\eta^{\mu\nu}, \quad [\alpha_n^{\mu}, \alpha_m^{\nu}] = \eta^{\mu\nu} n\delta_{n+m,0}, \quad \{c_n, b_m\} = \delta_{n+m,0}$$

Base vectors $|s, p\rangle = |f, p\rangle$ or $c_0 |f', p\rangle$ $|f, p\rangle = \alpha_{-n_1}^{\mu_1} \cdots \alpha_{-n_i}^{\mu_i} c_{-l_1} \cdots c_{-l_j} b_{-m_1} \cdots b_{-m_k} |0, p; \downarrow\rangle$

Action

$$S = -\frac{1}{2} \langle \Phi_1, Q \Phi_1 \rangle - \frac{g}{3} \langle \Phi_1, \Phi_1 * \Phi_1 \rangle$$

where Q is BRST operator (first quantized BRST charge)

$$Q = c_0 L_0 + b_0 M + \tilde{Q}$$

 $\langle \Phi, \Psi \rangle = \langle bpz(\Phi) | \Psi \rangle$

 $(\Phi * \Psi)[X(\sigma)] \sim \int DY DZ \Phi[Y(\sigma')] \Psi[Z(\sigma'')] \delta[X(\sigma), Y(\sigma'), Z(\sigma'')]$

Support of
$$\delta[X(\sigma), Y(\sigma'), Z(\sigma'')] \rightarrow X(\sigma)$$

 $Y(\sigma')$

Gauge invariance

Action \boldsymbol{S} is invariant under the gauge transformations

$$\delta \Phi_1 = Q \Lambda_0 + g(\Phi_1 * \Lambda_0 - \Lambda_0 * \Phi_1)$$

where $\Lambda_0 = \Lambda_0[X^{\mu}(\sigma), c(\sigma), b(\sigma)]$ is gh# 0 string field.

$$\Lambda_{0} = \int \frac{d^{26}p}{(2\pi)^{26}} \left[\lambda(p)b_{-1} + \cdots\right] |0, p\rangle$$
$$\rightarrow \delta A_{\mu}(p) = ip_{\mu}\lambda(p)$$

SU(1,1)

$$Q = c_0 L_0 + b_0 M + \tilde{Q}$$

$$M = -2\sum_{n>0} nc_{-n}c_n$$

M together with

$$M^- = -\sum_{n>0} \frac{1}{2n} b_{-n} b_n$$

and

$$M_{z} = \frac{1}{2}\widetilde{N}^{g} = \frac{1}{2}\sum_{n>0}(c_{-n}b_{n} - b_{-n}c_{n})$$

constitute SU(1,1) algebra

 $[M, M^{-}] = 2M_z, \quad [M_z, M] = M, \quad [M_z, M^{-}] = -M^{-}$

Isomorphism

$$\mathcal{F} = \bigoplus_{\widetilde{N}^g = -\infty}^{\infty} \left(\mathcal{F}^{\widetilde{N}^g} + c_0 \mathcal{F}^{\widetilde{N}^g} \right).$$

$$\begin{split} |f\rangle \in \mathcal{F}^{-n} \Rightarrow M^n |f\rangle \in \mathcal{F}^n, \quad M^n |f\rangle &= 0 \Rightarrow |f\rangle = 0 \\ \forall |f\rangle \in \mathcal{F}^n \Rightarrow \exists |g\rangle \in \mathcal{F}^{-n} \quad \text{s.t.} \quad |f\rangle = M^n |g\rangle \end{split}$$

There exists $W_n : \mathcal{F}^n \to \mathcal{F}^{-n}$ $W_n M^n | f \rangle = | f \rangle$ for any $| f \rangle \in \mathcal{F}^{-n}$ $M^n W_n | g \rangle = | g \rangle$ for any $| g \rangle \in \mathcal{F}^n$ $\mathcal{F}^n \stackrel{W_n}{\longleftrightarrow} \mathcal{F}^{-n}$ M^n

Gauge invariant decomposition

Expand string field in c_0

$$\Phi_1 = \phi^{(0)} + c_0 \,\omega^{(-1)}$$

In terms of $\phi^{(0)}$ and $\omega^{(-1)}$,

$$S_{2} = -\frac{1}{2} \left(\langle \phi^{(0)}, c_{0}L_{0}\phi^{(0)} \rangle + 2 \langle \tilde{Q}\phi^{(0)}, c_{0}\omega^{(-1)} \rangle + \langle M\omega^{(-1)}, c_{0}\omega^{(-1)} \rangle \right)$$
$$= -\frac{1}{2} \left\langle \left(\phi^{(0)} - \frac{1}{L_{0}}\tilde{Q}\omega^{(-1)} \right), c_{0}L_{0} \left(\phi^{(0)} - \frac{1}{L_{0}}\tilde{Q}\omega^{(-1)} \right) \right\rangle$$

Note that

$$\zeta^{(1)} = \tilde{Q}\phi^{(0)} + M\omega^{(-1)}$$

is gauge invariant for g = 0.

Using
$$\zeta^{(1)}$$
 instead of $\omega^{(-1)}$, we have

$$S_2 = -\frac{1}{2} \Big(\langle \phi^{(0)}, c_0 L_0 \phi^{(0)} \rangle - \langle \tilde{Q} \phi^{(0)}, c_0 W_1(\tilde{Q} \phi^{(0)}) \rangle + \langle \zeta^{(1)}, c_0 W_1 \zeta^{(1)} \rangle \Big)$$

Cf.
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \frac{1}{2}(A_{\mu}\partial \cdot \partial A^{\mu} - \partial \cdot A\partial \cdot A)$$

Siegel gauge

$$b_0 \Phi_1 = 0$$

Introducing ghost Φ_0 , anti-ghost Φ_2 and Nakanishi-Lautrup field \mathcal{B}_3 , gauge-fixed action is

$$S' = -\frac{1}{2} \langle \Phi_1, Q \Phi_1 \rangle - \frac{g}{3} \langle \Phi_1, \Phi_1 * \Phi_1 \rangle$$
$$- \langle \Phi_2, Q \Phi_0 + g \Phi_1 * \Phi_0 \rangle + \langle b_0 \mathcal{B}_3, \Phi_1 \rangle$$

Now we have another gauge symmetry

$$\delta \Phi_0 = Q \Lambda_{-1} + g(\Phi_0 * \Lambda_{-1} + \Lambda_{-1} * \Phi_0)$$

Ghost (for ghost) n

Again we fix

 $b_0 \Phi_0 = 0$

introducing "ghost for ghost" Φ_{-1} (and $\Phi_3,~\mathcal{B}_4)$

Again we have another gauge symmetry

Again gauge-fixing with "ghost for ghost for ghost $\cdots \cdots$ " $\Phi_0, \Phi_{-1}, \Phi_{-2}, \cdots$ $\Phi_2, \Phi_3, \Phi_4, \cdots$ $\mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_5, \cdots$

Procedure

1 Extend to all ghost number (field gh# + ws gh# = 1)

$$\Phi_1 \quad \rightarrow \quad \Phi = \sum_n \Phi_n$$

2 Find gauge conditions for all sectors

 $b_0 \Phi_n = 0$

s.t. (a) any string field can be transformed to satisfy them (b) no residual gauge symmetry shown for g = 0

3 Write gauge-fixed action

$$S_{\text{Siegel}} = -\frac{1}{2} \langle \Phi, Q\Phi \rangle - \frac{g}{3} \langle \Phi, \Phi * \Phi \rangle + \langle b_0 \mathcal{B}, \Phi \rangle$$

4 Check BRS invariance

 $\delta_B S_{\mathsf{Siegel}} = \mathbf{0}$

BRS transformation for Siegel gauge

$$\begin{split} \delta_B \Phi_n &= \eta \, b_0 \mathcal{B}_{n+1} & (n > 1), \\ \delta_B \Phi_n &= \eta \left(Q \Phi_{n-1} + g \sum_{k=-\infty}^{\infty} (\Phi_{n-k} * \Phi_k) \right) & (n \le 1), \\ \delta_B \mathcal{B}_n &= 0 \end{split}$$

 η is a grassmann odd parameter.

New covariant gauges

$$b_0(M + a c_0 \tilde{Q})\Phi_1 = 0$$

Gauge condition for all ghost number sector

For $n \ge 2$ $(b_0 M^{n-1} + a b_0 c_0 M^{n-2} \tilde{Q}) \Phi_{3-n} = 0$ $(b_0 W_{n-2} + a b_0 c_0 W_{n-1} \tilde{Q}) \Phi_n = 0$

a = 1 is prohibited for g = 0, since

 $(b_0 M + a b_0 c_0 \tilde{Q}) \Phi_1 = 0 \Leftrightarrow M \omega^{(-1)} + a \tilde{Q} \phi^{(0)} = 0$ L.H.S. reduces to (g = 0) gauge invariant combination at a = 1

$$\zeta^{(1)} = \tilde{Q}\phi^{(0)} + M\omega^{(-1)}$$

Feynman-Siegel point (a = 0)

$$b_0 M^{n-1} \Phi_{3-n} = 0 \iff b_0 \Phi_{3-n} = 0$$

$$b_0 W_{n-2} \Phi_n = 0 \iff b_0 \Phi_n = 0 \qquad (n \ge 2)$$

Because of the isomorphism between \mathcal{F}^n and \mathcal{F}^{-n}

$$b_0 M^{n-1} \omega^{(1-n)} = 0 \iff b_0 \omega^{(1-n)} = 0$$
$$b_0 W_{n-2} \omega^{(n-2)} = 0 \iff b_0 \omega^{(n-2)} = 0$$

$$\Phi_n = \phi^{(n-1)} + c_0 \omega^{(n-2)}$$

Gauge fixed action

$$S_{a} = -\frac{1}{2} \sum_{n=-\infty}^{\infty} \left\langle \Phi_{n}, Q\Phi_{-n+2} \right\rangle - \frac{g}{3} \sum_{l+m+n=3} \left\langle \Phi_{l}, \Phi_{m} * \Phi_{n} \right\rangle$$
$$+ \sum_{n=2}^{\infty} \left(\left\langle (\mathcal{O}_{a}\mathcal{B})_{-n+3}, \Phi_{n} \right\rangle + \left\langle (\mathcal{O}_{a}\mathcal{B})_{n}, \Phi_{-n+3} \right\rangle \right)$$

where

$$(\mathcal{O}_{a}\mathcal{B})_{n} = \left(b_{0}M^{n-1} + ac_{0}b_{0}M^{n-2}\widetilde{Q}\right)\mathcal{B}_{3-n},$$

$$(\mathcal{O}_{a}\mathcal{B})_{-n+3} = \left(b_{0}W_{n-2} + ac_{0}b_{0}W_{n-1}\widetilde{Q}\right)\mathcal{B}_{n}.$$

 S_a is invariant under the BRS transformation

$$\delta_{B} \Phi_{n} = \eta(\mathcal{O}_{a} \mathcal{B})_{n} \quad (n > 1)$$

$$\delta_{B} \Phi_{n} = \eta(Q \Phi_{n-1} + g \sum_{k=-\infty}^{\infty} (\Phi_{n-k} * \Phi_{k})) \quad (n \le 1)$$

$$\delta_{B} \mathcal{B}_{n} = 0$$

provided $\langle (\mathcal{O}_a \mathcal{B})_n, (\mathcal{O}_a \mathcal{B})_{-n+3} \rangle = 0.$

Landau point $a = \infty$

$$b_0 c_0 \tilde{Q} \Phi_1 = 0 \quad (\Leftrightarrow \tilde{Q} \phi^{(0)} = 0)$$

$$S = -\frac{1}{2} \left(\langle \phi'^{(0)}, c_0 L_0 \phi'^{(0)} \rangle + \langle M \omega^{(-1)}, c_0 \omega^{(-1)} \rangle \right) - \frac{g}{3} \left(\langle \phi'^{(0)}, \phi'^{(0)} * \phi'^{(0)} \rangle + 3 \langle \phi'^{(0)}, \phi'^{(0)} * c_0 \omega^{(-1)} \rangle + 3 \langle \phi'^{(0)}, c_0 \omega^{(-1)} * c_0 \omega^{(-1)} \rangle \right)$$

Remark: Banks-Peskin

$$\mathcal{L}_{\mathsf{BP}} = \frac{1}{2} \Phi(L_0 - 1) P \Phi$$

with projector P onto $L_n \Phi = 0$ (n = 1, 2, ...)Cf. IKKO (1985)

Level 1 (massless) fields

$$\phi^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{\alpha'}} \left(\gamma(p)b_{-1} + A_{\mu}(p)\alpha_{-1}^{\mu} + i\bar{\gamma}(p)c_{-1} \right) |0, p; \downarrow\rangle,$$

$$\omega^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{2}} \left(i\chi(p)b_{-1} + u_{\mu}(p)\alpha_{-1}^{\mu} + v(p)c_{-1} \right) |0, p; \downarrow\rangle.$$

$$\mathcal{B}_{\omega}^{N=1} = \int \frac{d^{26}p}{(2\pi)^{26}} \frac{1}{\sqrt{2}} \left(i\beta_{\chi}(p)b_{-1} + \beta_{u_{\mu}}(p)\alpha_{-1}^{\mu} + \beta_{v}(p)c_{-1} \right) |0, p; \downarrow \rangle.$$

$$S_{N=1}^{\text{quad}} = \int d^{26}x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\chi + \partial_{\mu} A^{\mu})^2 - i\bar{\gamma} \partial_{\mu} \partial^{\mu} \gamma - iu_{\mu} \partial^{\mu} \gamma \right]$$
$$+ \beta_{\chi} (\chi + a \partial_{\mu} A^{\mu}) + \frac{1}{2} \beta_{u\mu} (u^{\mu} - a \partial^{\mu} \bar{\gamma}) + \frac{1}{4} \beta_{v} v \right]$$

By use of field redefinitions

$$B = (a-1)\beta_{\chi} \qquad \tilde{\chi} = \chi + \partial_{\mu}A^{\mu} - \beta_{\chi}$$

$$\bar{c} = (a-1)\bar{\gamma} \qquad \tilde{u}^{\mu} = u^{\mu} - a\partial^{\mu}\bar{\gamma}$$

$$c = \gamma \qquad \qquad \tilde{\beta}_{u_{\mu}} = \beta_{u_{\mu}} + 2i\partial_{\mu}\gamma,$$

the above action can be written into the well-known form plus decoupled auxiliary fields' term

$$S_{N=1}^{\text{quad}} = \int d^{26}x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + B \partial_{\mu} A^{\mu} + \frac{\alpha}{2} B^2 + i\bar{c} \partial_{\mu} \partial^{\mu} c \right]$$
$$-\frac{1}{2} \tilde{\chi}^2 + \frac{1}{2} \tilde{\beta}_{u\mu} \tilde{u}^{\mu} + \frac{1}{4} \beta_v v \right]$$

with

$$\alpha = \frac{1}{(a-1)^2}$$

Gauge independence of on-shell amplitude

Propagator

$$\Delta_{a} = \frac{b_{0}}{L_{0}} - \frac{a}{1-a} \left(Q \frac{b_{0}c_{0}W_{1}}{L_{0}} + \frac{c_{0}b_{0}W_{1}}{L_{0}}Q \right) + \frac{a(2-a)}{(1-a)^{2}} Q \frac{b_{0}W_{1}}{L_{0}^{2}}Q$$
$$\Delta_{\text{Siegel}} = \frac{b_{0}}{L_{0}}$$
$$\Delta_{\text{Landau}} = \frac{b_{0}}{L_{0}} (1-P_{0}) + c_{0}W_{1}$$

Theorem

(I)
$$Q\Phi_i = 0, \ \Phi_i$$
: odd $(i = 1, \dots, n)$
 $\exists j \in \{1, \dots, n\}$ s.t. $\Phi_j = QZ$ (Z: even
 $\implies A_{\text{cyclic}}^{\text{tree}}(\Phi_1, \dots, \Phi_n) = 0$

(II)
$$Q\Phi_i = 0, \ \Phi_i$$
: odd $(i = 1, \dots, n)$
 $\implies A_{\text{cyclic}}^{\text{tree}}(\Phi_1, \dots, \Phi_n)_{\Delta_a} = A_{\text{cyclic}}^{\text{tree}}(\Phi_1, \dots, \Phi_n)_{\Delta_0}$

Application to tachyon condensation

Examining gauge (in-)dependence of tachyon potential

Sen's conjecture



Potential depth of the vacuum is equal to D25-brane tension T_{25} .

No open string excitation around the tachyon vacuum.

Lower dimensional D-branes are solitonic solutions around the vacuum.

Tachyon potential in level truncation

We take twist-even, $N^g = 1$, scalar fields up tp level L

$$\Phi_1^{(L)} = \phi |\downarrow\rangle + \sum_i \phi_i |f_i\rangle + c_0 \sum_j \omega_j |g_j\rangle$$

Level truncated potential

$$V^{(L,3L)}(\phi, \{\phi_i\}, \{\omega_j\}) = -S\left(\Phi_1^{(L)}\right).$$

Gauge fixing

we substitute $\omega_j = \omega_j(\{\phi_i\})$ for $|a| < \infty$ or ϕ'_j for $a = \infty$ where $\{\phi'_j\}$ is a set of solutions to $\tilde{Q}\phi^{(0)} = 0$

Example

Fields up to Level 2:

 $\Phi_1^{(L=2)} = \phi |\downarrow\rangle + \phi_1(\alpha_{-1} \cdot \alpha_{-1}) |\downarrow\rangle + \phi_2 b_{-1} c_{-1} |\downarrow\rangle + \omega_1 c_0 b_{-2} |\downarrow\rangle.$

Gauge condition:

$$-4\omega_1 + a(26\phi_1 + 3\phi_2) = 0$$

Level (2,6) truncated potential:

$$V_a^{(2,6)}(\phi,\phi_1,\phi_2) = V^{(2,6)}(\phi,\phi_1,\phi_2,\omega_1 = a(26\phi_1 + 3\phi_2)/4),$$

$$V_{\infty}^{(2,6)}(\phi,\phi_1,\omega_1) = V^{(2,6)}(\phi,\phi_1,\phi_2 = -26\phi_1/3,\omega_1).$$

Tachyon potential at varying *a*



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Dangerous zone

Fields up to Level 2:

$$\Phi_1^{(L=2)} = \phi |\downarrow\rangle + \phi_1(\alpha_{-1} \cdot \alpha_{-1})|\downarrow\rangle + \phi_2 b_{-1} c_{-1}|\downarrow\rangle + \omega_1 c_0 b_{-2}|\downarrow\rangle.$$

Gauge transformation

$$\begin{split} \delta \phi &= g \bar{\kappa} \lambda \left(-\frac{16}{9} \phi + \frac{2080}{243} \phi_1 - \frac{464}{243} \phi_2 - \frac{128}{81} \omega_1 \right), \\ \delta \phi_1 &= \frac{\lambda}{2} + g \bar{\kappa} \lambda \left(\frac{40}{243} \phi - \frac{9296}{6561} \phi_1 + \frac{1160}{6561} \phi_2 + \frac{320}{2187} \omega_1 \right), \\ \delta \phi_2 &= -3\lambda + g \bar{\kappa} \lambda \left(-\frac{176}{243} \phi + \frac{22880}{6561} \phi_1 - \frac{11248}{6561} \phi_2 + \frac{6016}{6561} \omega_1 \right), \\ \delta \omega_1 &= \lambda + g \bar{\kappa} \lambda \left(\frac{224}{81} \phi - \frac{29120}{2187} \phi_1 - \frac{992}{6561} \phi_2 + \frac{1792}{729} \omega_1 \right), \\ \text{where } \bar{\kappa} = \frac{1}{3} \left(\frac{3\sqrt{3}}{4} \right)^3 \end{split}$$

At around a = 1.85 gauge slice is parallel to the gauge flow through the vacuum of gauge-unfixed action. (cf. a = 1 for g = 0)

Branch structure



Level(0,0) black

Level(2,6) $a = \infty$ (Landau) orange

Level(2,6) a = 0 (Feynman-Siegel) blue

Higher level







Vacuum solution



Level(2,6) sky-blue

Level(4,12) blue

Level(6,18) rose



Level(2,6) sky-blue Level(4,12) blue Level(6,18) rose

	Level			
	(0,0)	(2,6)	(4,12)	(6,18)
a	$E_{\rm Vac}/T_{25}$	$E_{\rm Vac}/T_{25}$	$E_{\rm Vac}/T_{25}$	$E_{\rm Vac}/T_{25}$
∞	-0.68462	-0.91328	-0.94758	-0.96094
4.0		-0.88520	-0.91189	-0.92449
0.5		-0.97704	-1.00030	-1.00459
0.0		-0.95938	-0.98782	-0.99518
-2.0		-0.93477	-0.96842	-0.97989

Conclusions

Summary

New covariant gauges are proposed

A single parameter family of gauges which naturally corresponds to the covariant gauges in ordinary gauge theory.

Action is simplified in Landau point.

Applied to tachyon potential

The identification of physical branch.

Branching behavior in Siegel gauge is gauge-artifact.

Outlook

Analysis of space(-time) dependent solutions may be also simplified in Landau point (both numerically and analytically.)

Absence of open string mode on tachyon vacuum: Kishimoto-Takahashi (2002), Imbimbo (hep-th/0611343) ↔ Ellwood-Schnabl (hep-th/0606142)

Non-perturbative gauge fixing ambiguity

Relation to Feng-Siegel (hep-th/0611307)