

**Mathematical Climatology Laboratory (2021)**  
**Chief Scientist: Hirofumi Tomita (D.Eng.)**



**(0) Research field**

CPR Subcommittee: Physics and Engineering

**Keywords:** Self-organization and hierarchical structure of cloud / Multi-Equilibrium Solution of climate / Development of regional climate assessment method

**(1) Long-term goal of laboratory and research background**

From the mathematical viewpoint, we clarify the essential mechanism, such as the self-organization of clouds and their hierarchical structure, based on the understanding of the processes of clouds, turbulence, and radiation that are important for climate change. Based on these understandings, we will develop mathematical methods for climate assessment while interpreting the uncertainty of future climate projection. Through this, we aim to make a direct contribution to society. In the former essential subject, we evaluate the convergence of the solution of the cloud and turbulence schemes and aim to construct the theory of Large Eddy Simulation (LES), including the phase change of water. In addition, our aim includes the improvement of the physical processes and the development of high-speed algorithms for those schemes. Finally, it leads to the proposal of appropriate parameterization for low-resolution models. We will apply the new climate assessment methods we have developed to various problems in the latter subject. As one concrete theme, we will downscale the forecast results of the global climate projected by each model. Thus, it will allow us to understand the uncertainty and evaluate the future climate in each region with more objectivity.

**(2) Current research activities (FY2021) and plan (until Mar. 2025)**

**1. A mechanism of self-aggregation onset of moist convection in radiative-convective equilibrium**

Radiative-convective equilibrium (RCE) is an idealized depiction of the atmospheric energy balance and has long been studied as a framework for exploring the essential properties of the climate system. In recent years, RCE studies using cloud-resolving models that can explicitly treat convective clouds and associated atmospheric vertical motion have been actively pursued worldwide. However, despite the fact that RCE experiments are regarded as a simple framework among various idealized numerical climate experiments, the full picture of the rich spatio-temporal structure emerged by the self-organization of moist convection is still unclear. In particular, it has become clear that the large-scale organization of moist convection strongly depends on the domain size and resolution used in RCE experiments (Muller and Held 2012; Patrizio and Randall 2019). We have been conducting systematic RCE experiments using the numerical weather and climate library SCALE (Nishizawa et al. 2015; Sato et al. 2015), which is being developed mainly by the Computational Climate Science Research Team at the RIKEN Center for Computational Science, and have produced pioneering results in this research field by clarifying the conditions for the onset of self-aggregation of moist convection with respect to domain size and resolution (Yanase et al. 2020).

This year, we further expanded on the work done up to the previous year to study the onset mechanism of cloud self-aggregation based on a detailed analysis of the dependence of the RCE experiment on the domain size. The analysis focused on the role of circulation in the lower atmosphere and horizontal variability of radiation, convection, and water vapor in the upper atmosphere in the spontaneous amplification of the horizontal contrast in atmospheric water vapor, which is a prominent feature of cloud self-aggregation in RCE. First, we found that a characteristic low-level circulation develops when the domain size is sufficiently large and aggregation occurs, unlike the case where the domain size is small and aggregation does not occur. The low-level circulation is responsible for the upgradient horizontal water vapor transport. The driving force of this low-level circulation is a buoyancy gradient generated by strong radiative cooling in the dry region. This driving mechanism of circulation by radiative cooling explains the development of self-aggregation on relatively long time scales of several tens of days. On the other hand, horizontal variability in the upper atmosphere is important for the triggering mechanism of self-aggregation on shorter time scales prior to the development of the low-level circulation. Many observational and numerical studies have shown that an increase in atmospheric water vapor content is associated with an increase in atmospheric convective heating. This relationship was reproduced in a series of experiments in this study; in particular, the range of their variations increases as the domain size increases. It is suggested that this increase in variability leads to an intensification of subsidence in the dry region, triggering self-aggregation through vertical penetration of the air from the upper to the lower atmosphere. The horizontal variability of water vapor content and convection is closely related to their spatial scales, and the representation of large spatial scale variability with increasing domain size is key to understanding the domain size dependence of the onset of self-

aggregation. Finally, based on these results, we present a new picture of the mechanism of self-aggregation onset that integrates processes in the upper and lower atmosphere (Figure 1). The results of this research are being submitted to the Journal of the Atmospheric Sciences.

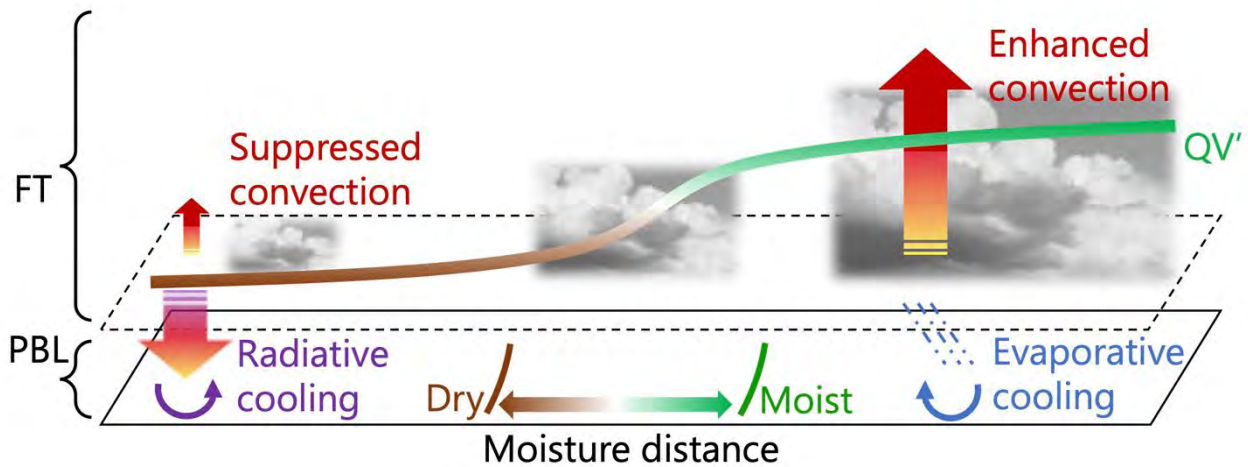


Figure 1. Schematic of the onset of self-aggregation of moist convection. From Figure 18 of Yanase et al. (2022, *J. Atmos. Sci.*, under review).

So far, we have studied the mechanism of self-aggregation on spatial scales of a few hundred to 1,000 km. In the future, in order to clarify how self-aggregated clouds form higher-level hierarchical structures on larger spatial scales of several 1,000 to 10,000 km, we plan to conduct an unprecedented extremely large-domain RCE experiment using the supercomputer "Fugaku" to further explore cloud self-organization and hierarchical structures.

## 2. Derivation of required numerical accuracy for turbulence scheme LES

Recently, large eddy simulation (LES) has begun to be employed in weather and climate simulations as a promising method for sub-grid-scale turbulence parameterization. However, the numerical accuracy required for dynamical cores is still not fully understood. Last year, we derived two theoretical criteria for the required accuracy of the advection term and verified their validity by numerical experiments setting typical conditions of the atmospheric boundary layer. Based on the derived numerical criteria, we found that at least the 7th or 8th order is required for a grid spacing of  $O(10\text{m})$  (Kawai & Tomita, 2021). However, to achieve such a high order in the grid-point method, a large amount of communication between parallel computers is required, which degrades computational efficiency. The Discontinuous Galerkin method (DGM) is a promising approach to overcoming these problems. For this reason, in this fiscal year, we have derived a theoretical numerical criterion for DGM in LES and clarified the required degree of the polynomial ( $p$ ) in the element. Based on numerical criteria with a grid spacing of  $O(10\text{ m})$ , we found that  $p = 4$  is necessary, along with a sufficiently scale-selective modal filter. For temporal accuracy, it was suggested that the 4th order is sufficient if a fully explicit temporal scheme was used. We also investigated the effect of hyper-upwinding, which is commonly seen when the Rusanov flux is used in low Mach number flows. As a result, it was found that the choice of numerical flux had little effect on the simulation results when high-order DGM was used. A series of these theoretical conclusions were validated using actual boundary layer turbulence simulations. Two papers are currently in preparation for submission. As a further subject, we will construct a global model using DGM.

## 3. Model parameter estimation using data assimilation

Various parameters are included in the physical process components in the climate model. These parameters have uncertainties and become factors of uncertainty in climate prediction. To improve the reliability of climate prediction, we aim to establish a method for estimating optimal parameter values by data assimilation. In this fiscal year, we regarded the results of a numerical simulation of a moist convection system as observations (nature run) and carried out an ideal experiment to estimate the true values of the parameters used in the nature run by data assimilation using the ensemble Kalman filter (EnKF). EnKF-based methods need to apply some inflation to the ensemble spread (ES) of the parameter values to preserve the uncertainty margin of the parameters to prevent divergence of the estimated parameters. Here, when

applying inflation in which the magnitude of the parameter value ES is kept constant throughout the estimation period, the accuracy of estimation and the convergence speed depends on the magnitude of ES. However, there is no consensus on how large ES should be set to optimize estimation accuracy and convergence speed. From the results of the above experiments, it was found that the smaller the ES, the higher the estimation accuracy, but there is a lower limit of ES beyond which accuracy cannot be expected, and we can determine the optimum ES from the viewpoint of accuracy. On the other hand, the convergence speed increases as ES increases. We approximate the parameter estimation time series by a first-order autoregressive model (AR(1)), and formulate the estimation accuracy and convergence speed from two parameters of the AR(1) model, i.e., the autoregressive parameter and the random disturbance amplitude. As ES increases, the autoregressive parameter decreases and the random disturbance amplitude increases. Estimation accuracy is determined by a balance between the two. The AR(1) model approximation is considered to be a quantitative guideline in determining the ES that optimizes the accuracy of parameter estimation and convergence speed. This year, we will submit these results to PEPS, which have been accepted as of September 2022. In the future, we plan to conduct experiments in the real atmosphere and demonstrate the usefulness of this subject.

### Precision and Convergence Speed of EnKF-Based Parameter Estimation

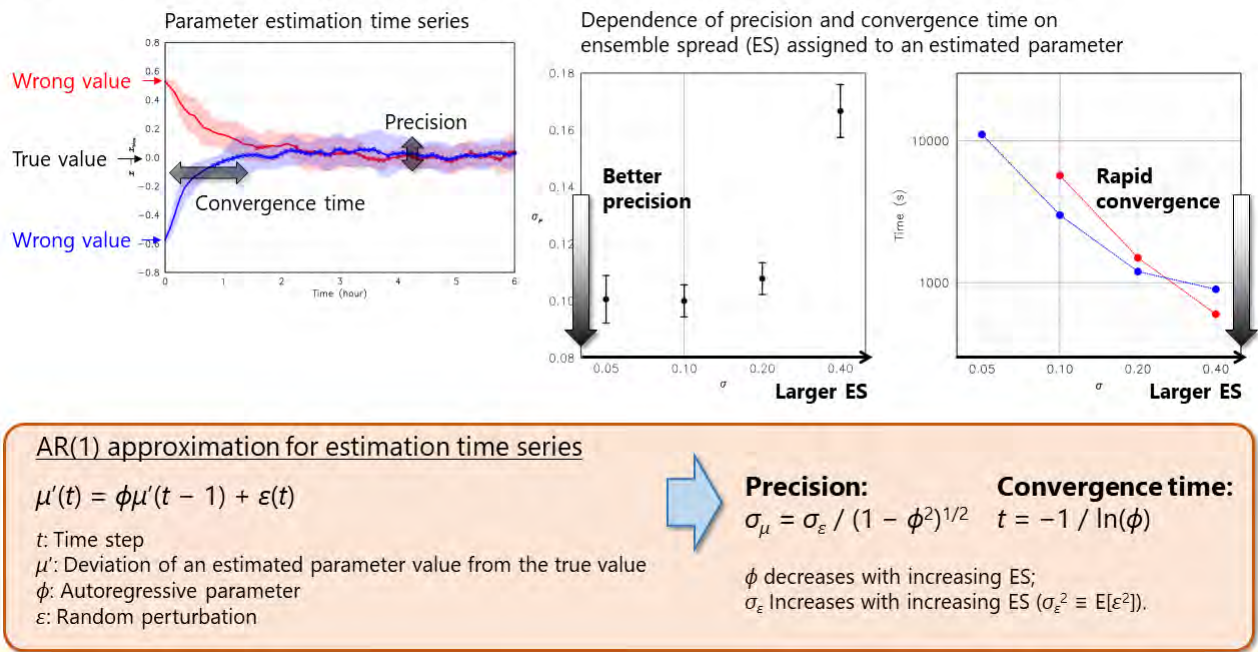


Fig.2: Left panel: Time series of the parameter estimation where the initial parameter value is larger than the true value (red line) and it is smaller than the true value (blue line). Color shade represents the standard deviation of 13 trials for each setting; the smaller the standard deviation, the higher the estimation precision. As for the convergence speed, the shorter the time required for the estimated value to converge around the true value, the faster the convergence speed. Middle panel: Dependence of the estimation precision on the ensemble spread (ES) of the estimated parameter. Right panel: Dependence of the convergence time on the parameter ES. Description in the bottom box: Formulation of the estimation precision and convergence time using the autoregressive parameter  $\phi$  and the amplitude of random perturbation  $\varepsilon$  when the estimation time series is approximated as the AR(1) model. (from Sueki et al. 2022 )

### (3) Members

(Chief Scientist)

Hirofumi Tomita

(Research Scientist)

Sachiho A. Adachi

(Postdoctoral Researcher)

Kenta Sueki

FY2021

### (4) Representative research achievements

1. Kawai, Y. and H. Tomita (2021): "Numerical Accuracy of Advection Scheme Necessary for Large-Eddy

Simulation of Planetary Boundary Layer Turbulence”, *Mon. Wea. Rev.*, 149(9),  
<https://doi.org/10.1175/MWR-D-20-0362.1>.

2. Sato, Y., Y. Miyamoto, and H. Tomita (2021): “Lightning frequency in an idealized hurricane-like vortex from initial to steady-state using a coupled meteorological and explicit bulk lightning model”, *Mon Wea Rev.*, 49(3), Doi:10.1175/MWR-D-20-0110.1.
3. Tomita, H. : “The co-design of general-purpose supercomputer "Fugaku" and future of computational climate study”, *The Fifth Convection-Permitting Modeling Workshop 2021*, Sept. 10, 2021, Online (invited)
4. Adachi, S.A.: “Review of dynamical downscaling techniques for assessing regional climate change”, *The Fifth Convection-Permitting Modeling Workshop 2021*, Sept. 10, 2021, Online.
5. Yanase, T., Nishizawa, S., Miura, H., Takemi, T., Tomita, H. “New Critical Length for the Onset of Self-Aggregation of Moist Convection”, *The Fifth Convection-Permitting Modeling Workshop 2021*, Virtual, Sep, 2021. (Poster)

## **Supplementary**

### **Laboratory Homepage**

[https://www.riken.jp/research/labs/chief/math\\_clim/index.html](https://www.riken.jp/research/labs/chief/math_clim/index.html)

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